Vol. XL. DECEMBER, 1898.

AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

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No. 839.

RESERVOIR SYSTEM OF THE GREAT LAKES OF THE ST. LAWRENCE BASIN;

ITS RELATION TO THE PROBLEM OF IMPROVING THE NAVIGATION OF THESE BODIES OF WATER AND OF THEIR CONNECTING CHANNELS.

By HIRAM M. CHITTENDEN.*

WITH A MATHEMATICAL ANALYSIS OF THE INFLUENCE OF RESERVOIRS UPON STREAM FLOW.

By JAMES A. SEDDON†, M. Am. Soc. C. E.

PRESENTED AT THE ANNUAL CONVENTION, JULY, 1898.

WITH DISCUSSION.

The immense system of inland navigation, known as the Great Lakes of the St. Lawrence Basin, possesses many features in common with the navigation of the high seas. These bodies of water are of great magnitude. The vessels which sail upon them rival in size sea-going craft, and are built to withstand storms which approach in fury those of the ocean itself. Light-houses and fog signals define the position of dangerous coasts, and breakwaters are required to protect the open roadsteads. The lake ports are essentially maritime ports, and the connecting channels resemble maritime straits and canals

* Captain, Corps of Engineers, U. S. A.: Lieutenant-Colonel and Chief Engineer, U. S. Volunteers.

† U. S. Assistant Engineer.

more than they do ordinary rivers. The commerce of the lakes has now reached a magnitude which makes a respectable comparison with the ocean commerce of various parts of the world. Several of the lake ports do a business exceeded by but few seaports, and the commerce which passes the Sault Ste. Marie Canal and the St. Clair and Detroit rivers surpasses in volume that of the world's greatest maritime canal. Similarity of conditions has given rise to similarity of problems, such as the protection of ports, the deepening of harbors, and the enlargement of connecting channels, which are common objects in the interests of navigation, alike on the lakes and on the ocean.

There are, however, two important physical characteristics which create a notable difference between the conditions of ocean and lake navigation. One of these is the existence of diurnal tides on the ocean which presents problems of a distinct character not met with except in tidal waters. The other belongs to the lakes, and relates to their condition as bodies of fresh water with a perennial overflow. This characteristic not only gives rise to problems new and distinct in themselves, but it affords new solutions for old problems pertaining to maritime navigation in general. It is to a consideration of this particular feature of the navigation of the Great Lakes that the present paper is devoted.

Notwithstanding the vast magnitude of these lakes and their resemblance in many particulars to tidal waters, they are, after all, only parts of an immense river system which drains a large area of country. They are subject to the variable conditions of water supply characteristic of all streams. The connecting channels and the final outlet carry a continuous current, always in one direction, unlike maritime straits and canals through which the water may flow in either direction depending upon the state of the tide or of the wind. But while the lakes and their connecting channels thus constitute a great river system, that system differs from nearly all others in this particular—that the flow in the outlets is practically exempt from those variations and irregularities which obtain upon nearly all other streams.

The explanation of this most important characteristic is to be found, of course, in the controlling action of these lakes as great reservoirs interposed in the course of the stream. The storage represented by a few inches rise and fall in the lake levels is enormous, and when withheld during the flood season and released in the dry season, it gives to the outlets a regimen of flow which is radically dif-

ferent from that of an ordinary river. This influence is not annual merely, but cyclic as well, and a series of what may be called wet years causes a general rise of mean level, the storage of which maintains the flow in the outlets during the ensuing years of less than average precipitation. The balance of forces which Nature has here produced in the course of long ages is one of the most marvelous features of these lakes; and a careful contemplation of it cannot fail to convince one that an almost perfect compromise has been reached between the conflicting oscillations of lake level and outlet discharge. It is difficult to see how either could be brought nearer to absolute uniformity without a resulting departure in the other which would more than offset the gain.

Perfect as this natural condition is, however, it yet does not satisfy the ambition of those who make commercial use of it. There are many who consider the oscillation in lake levels as an evil which ought not to be suffered to continue. The ship-owner, who loads his boat to the full limit of harbor depth, complains when the waters subside and compel him to load to a lighter draft. The periodic oscillation in particular, which every now and then results in a continuous subsidence of mean level through a series of years, is very damaging to commercial interests and naturally causes a good deal of apprehension. Lake carriers are led, on account of it, to clamor for a corrective, which, if applied, might entail greater evils than those of a falling lake level, for they apparently forget that it is this very subsidence of level which keeps the water in the connecting channels from subsiding in a much greater degree.

Various causes have contributed to give this subject unusual prominence in recent years. A period of what might be called dry years culminated in 1895, in the lowest mean level in Lakes Michigan, Huron and Erie that has ever been experienced since the commerce of the lakes has been a matter of great moment. The artificial enlargement of the natural cross-sections of some of the connecting channels, for the improvement of navigation, has been thought by many to have contributed to this result. Simultaneously with the continuance of this unfavorable condition were the commencement and vigorous prosecution of a project which contemplates the permanent withdrawal of a large amount of water from Lake Michigan, the possible maximum of the diversion being 10 000 cu. ft. per second. All of these coincident

causes of alarm created no little uneasiness among commercial interests, and gave rise to a great deal of earnest consideration as to ways and means of averting the threatened dangers. Several public discussions and a considerable amount of professional literature on the subject were the first result. The matter finally came up in Congress in the shape of several reports which were made the basis of a bill* to provide for a thorough investigation of the whole question. In the meanwhile Nature commenced removing the immediate cause of apprehension, for the mean lake levels have been rising since 1895, and very likely will continue to rise for some time to come. As a natural result, the urgent clamor for action, which was so prominent a few years ago, subsided as the lake levels rose, and will probably not assume troublesome proportions again until a new period of low water arrives. The problem, nevertheless, is before us for solution, and the respite afforded by the natural relief now being enjoyed should not be permitted to go unimproved, but should be utilized to the utmost to bring about a well-digested plan of action before the hour of necessity again arrives.

This problem, as developed by the general discussions already referred to, aims to secure three objects:

(1) A control of the lake levels by which their annual and periodic oscillations may be, if not altogether eliminated, still materially reduced in range. This purpose is expressed in the following extract from the bill proposed to Congress in the winter of 1895–6, which directed, among other things, an inquiry and an investigation as to whether:

"it is practicable to control the waters of the Great Lakes and maintain them at substantially a uniform level at all seasons of the year, by a dam or dams or other works placed in Niagara River at the outlet of Lake Erie, and by a system of wing dams or other structures placed in the Detroit River and the St. Clair River, and the Sault Ste. Marie or St. Mary's River, at or near the respective outlets of Lakes Saint Clair, Huron and Superior."

The words "substantially a uniform level" are not specifically interpreted, but a limit of oscillation of 6 ins. has been urged as a practicable one, at least for Lake Erie.

(2) A permanent elevation of the lake levels by means of contracting works in the outlets. This, it is urged, would be the sim-

* This particular bill did not become a law.

plest, most effective and most economical method of securing a permanent increase of depth in the ports and connecting channels of the lakes.

(3) Some means of counteracting the tendency to a permanent lowering of lake levels due to an artifical enlargement of natural cross-sections of the connecting channels, or to the permanent diversion of any portion of the overflow to another water-shed.

It may be accepted as a preliminary condition, to which there can be no qualification, that any alteration of natural conditions in the Great Lakes, which shall result in a material diminution of flow in the outlets during any portion of the navigation season, cannot be considered for a moment. It therefore becomes a matter of the first importance to determine what effect the attainment of any of the above objects will have on the flow in these outlets.

Inasmuch as the navigation system of the Great Lakes, as already pointed out, is virtually a great river regulated by immense reservoirs distributed along its course, it is manifest that the hydraulic features of the problem are inseparably bound up with the laws of reservoir action in the regulation of stream flow.

Fundamental Propositions Pertaining to the Action of Reservoirs in the Regulation of Stream-Flow.

I.—*The effect of a natural reservoir upon a stream passing through it is to reduce variations and irregularities of flow and to give the stream a more uniform discharge upon leaving the reservoir than it had before entering.*

The explanation of this common phenomenon is obvious. When the inflow into a reservoir increases, the outflow will not generally increase at the same rate, for a portion of the inflow will be absorbed in storage due to a rise in the reservoir level. In like manner, when the inflow is decreasing, the outflow will not generally decrease at the same rate, for it will be reinforced by the storage which flows out as the reservoir falls. So that, while the total outflow may equal the total inflow, it will not generally show the same extremes of variation. The effect is analogous to that of a balance wheel upon the motion of machinery; the wheel being virtually a reservoir of force, in which irregular and sudden impulses are stored, and from which the stored force is paid out to sustain motion whenever there are sudden or irregular cessations in the application of the moving force.

To give mathematical expression to this proposition:

Let S represent supply in the most general sense of that term, positive and negative, including inflow from other reservoirs, rainfall, run-off, evaporation, and all other sources of supply or loss except the discharge of the outlet. It is expressed in second-feet and may have negative values.

Let Q represent the discharge of the outlet in second-feet.

Let h represent the variable height in feet of the reservoir surface above any datum, and also of the outlet immediately at the reservoir.

Let t represent the time in seconds from an assumed origin to any value of h.

Let A represent the area of the reservoir in square feet.

Then, at any instant, the rate of change in h will be represented by the following equation:

$$(S - Q)\, dt = A\, dh \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The equation for the discharge of the outlet is

$$Q = f(h) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which $f(h)$ always increases and diminishes with h, its particular form, in any given case, depending upon the character of the outlet.

From equation (1) it is seen that whenever h is increasing, dh is positive, and $Q < S$; and that whenever it is diminishing dh is negative and $Q > S$; also, that when h and Q are at maximum or minimum, dh is zero and $Q = S$.

In any cycle of changes, the maximum value of S precedes and is greater than the maximum value of Q. For if the curves of S and Q were both plotted to the same time scale, and if both maxima were equal and simultaneous, the two curves would be tangent to each other at that point where $Q = S$. But it has been seen that when Q is approaching its maximum it is less than S, and that after leaving its maximum it is greater than S. The two curves thus intersect at the point where $Q = S$, and therefore cannot be tangent; and since, at this point, Q is at a maximum and S is falling, S must have already passed a maximum greater than that of Q. In like manner, it will be seen that the minimum of Q follows, and is greater (algebraically) than that of S. The limits of oscillation in Q, therefore, lie within those of S.

From equation (1) it will also be seen that, other things being equal, the moderating effect of a reservoir upon stream flow varies directly with its area.

In the general proposition above stated the word "natural" was used by way of qualification, because in the practical uses to which artificial reservoirs are often put, the proposition is not strictly true. In some reservoirs the outlets are entirely closed during certain periods, and the water is all released at other periods, so that, in such cases, the natural fluctuations in the flow of the stream may actually be increased, or at least be radically altered in their periods.

Even in natural reservoirs there are apparent, if not real, exceptions. If a reservoir be very small in comparison to the volume of water flowing through it, but little larger in fact than the channel of the stream itself, the rise in the reservoir will follow so closely upon that in the stream that the two may appear to be simultaneous and equal. This is but one illustration of what may be stated as a general rule, that every stream is virtually a series of small reservoirs and connecting channels; and that all stream flow is more or less subject to the influence of reservoirs.

Reservoirs in Series.

As a corollary to the foregoing proposition, it follows that, if several reservoirs succeed each other in a descending series, so that a supply to any reservoir must flow successively through all below, the fluctuation of level in the different reservoirs, due to a variable supply to an upper reservoir, will diminish with a descent of the series and will approach zero as a limit.

It has been seen that in any reservoir the variation of Q is less than that of S, but, neglecting local supply, Q of one reservoir becomes S of that next below. Calling this S', and the discharge of the lower outlet, Q', there results:

$$Variation\ Q' < Variation\ S' < Variation\ S, \text{ etc.};$$

but $Variation\ h = \frac{Variation\ Q}{c}$ (see notation just below), in which c, as a rule in natural reservoirs, will increase with a descent of the series. The change in each term of the above ratio is therefore such as to diminish the value of $Variation\ h$*.

* The term "variation," as used in this paper, is not to be taken in its mathematical signification, but simply as denoting the maximum fluctuation in S and Q. It corresponds to the term "oscillation," as applied to h.

There are possible exceptions to this rule. If it should happen that a particular reservoir were of very small area, with an engorged outlet, in which the value of c were actually smaller than that in the outlet next above, the value of $\frac{Variation\ Q}{c}$ might be greater than for the next preceding reservoir. Examples of this are common in the flow of nearly all streams. When the channel of a river is engorged from any cause, the value of c is less at this point than either above or below. If the river valley above be considered as a very small reservoir, we shall have a true example of the above exception. It is known, as a matter of fact, that the rise of rivers in times of flood varies materially at different points of their course owing to this cause. But while this exception is true of such extreme cases as have been here cited, it would not be true of any ordinary system of reservoirs.

For a more specific study of these relations:

Let A denote the area of the reservoir in square miles.

Let h denote the height in feet of the surface of any reservoir above the mean level for the cycle. For the first reservoir considered, it will be written h_1; for the next reservoir the resulting oscillation will be h_2, and so on to h_n.

Let c denote the variation of Q in second-feet for a change of 1 ft. in h, $f(h)$ being assumed to be a right line.*

Let P denote the coefficient of reduction of S, or the ratio $\frac{Q\ (\text{maximum})}{S\ (\text{maximum})}$.

Let p denote the coefficient of reduction of h, or the ratio of h for one reservoir divided by h for that next above, or $\frac{h_n\ (\text{maximum})}{h_{n-1}\ (\text{maximum})}$.

Let R denote the interval which elapses between the maximum or minimum of S in any reservoir, and the resulting maximum or minimum of Q. R is expressed in degrees of arc; a full circle, or 360°, representing a complete cycle, taken in this case as one year.

Let the variations of supply be represented by the equation $S = a \sin t$, in which a is the extreme variation in second-feet above or below the mean, and t is degrees of arc, of which 360 make a complete cycle.

* In streams of considerable magnitude and small variation in discharge, the assumption that $f(h)$ is a right line for slight changes in h, involves so small an error as not to impair in the least degree the validity of the mathematical processes in which $f(h)$ enters.

Then the following equations may be written:*

$$\text{Sin } R_n = \frac{1}{\sqrt{1 + \left(\frac{0.18c_n}{A_n}\right)^2}} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

$$P_n = \frac{1}{\sqrt{\left(\frac{A_n}{0.18c_n}\right)^2 + 1}} \quad \ldots\ldots\ldots\ldots\ldots (4)$$

$$p_n = \frac{0.18c_{n-1}}{\sqrt{A^2{}_n + (0.18c_n)^2}} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

From these equations it is seen that, for a given cycle, R is a function of area and outlet alone, and independent of S; P also is a function of area and outlet alone; and p is a function of area and of inlet and outlet.

It will also be seen that R decreases as A increases, or as c of the outlet decreases; P increases as A decreases, or as c of the outlet increases; while p increases as A and c of the outlet decrease, and as c of the inlet increases. In each of the above cases the converse statement is true.

Inasmuch as the conditions which control the oscillation of level in natural reservoirs are seldom, if ever, such as to bring the crests of transmitted waves from upper reservoirs, and those due to local supply, simultaneously together, wave interference, which to some extent neutralizes the combined effect, is the result. In other words, the maximum oscillation of level in any reservoir of a series is never the sum of the maxima of local and transmitted oscillations, for these are never exactly superimposed; and they may even so occur that the hollow of the transmitted wave comes upon the crest of the local wave, and may actually cause a smaller oscillation than if there were no transmitted wave.

As a general proposition it may be said that, in a series of natural reservoirs, such as has been described, each with its independent local supply, the oscillation of levels will be mainly controlled by the transmitted supply when the reservoir area is relatively small, and by the local supply when its area is relatively large.

For the better illustration of the principles above enunciated, let the following example be considered. Let there be five reservoirs,

* See mathematical analysis hereto appended.

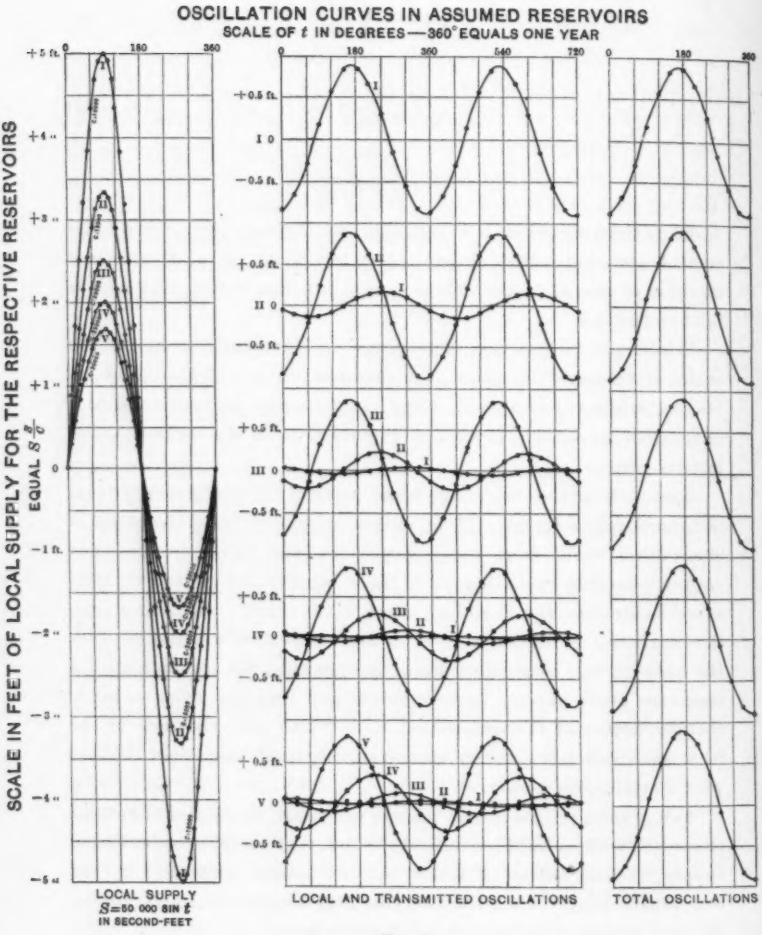

OSCILLATION CURVES IN ASSUMED RESERVOIRS
SCALE OF t IN DEGREES—360° EQUALS ONE YEAR
SCALE IN FEET OF LOCAL SUPPLY FOR THE RESPECTIVE RESERVOIRS EQUAL S g/c
LOCAL SUPPLY S=50 000 SIN t IN SECOND-FEET
LOCAL AND TRANSMITTED OSCILLATIONS
TOTAL OSCILLATIONS

FIG. 1.

and assume the area of each to be 10 000 square miles, the range of variation in S to be 100 000 second-feet, and to be the same for each of the reservoirs; and the values of c, beginning with the first reservoir, to be 10 000, 15 000, 20 000, 25 000 and 30 000 second-feet. The values of R, P and p, as computed from equations (3), (4) and (5), may then be written as follows:

Reservoir.	R.	P.	p.
No. 1	81 days.	0.177	.....
No. 2	76 "	0.261	0.174
No. 3	71 "	0.339	0.254
No. 4	67 "	0.410	0.328
No. 5	62 "	0.475	0.396

The accompanying oscillation curves (Fig. 1) show for each reservoir the element of total oscillations which comes from local supply, and that from each of the reservoirs above, and enables one to judge at once by the eye what proportion of the total oscillation in any case is due to local supply, and what to transmitted supply. In the example above given, in which the reservoirs are relatively large, the oscillation is mainly due to local supply. For instance, in the third reservoir, the maximum oscillation, due to local supply, is 95½% as great as the actual oscillation, and occurs 12 days earlier. The maximum transmitted oscillation is 25½% as great as the actual, and occurs 68 days later.

To show what would be the effect upon a comparatively small reservoir, suppose that in the above series, in place of the third unit, there is introduced a reservoir of the following size and characteristics: Area, 400 square miles; range of variation in S, 4 000 second-feet; c of outlet, 20 000 second-feet. It will then be found that the maximum of oscillation, due to local supply, is 30% as great as the actual, and occurs 67 days earlier; while the maximum of oscillation, due to transmitted supply, is 94% as great as the actual, and occurs 20 days later. In this reservoir, therefore, the oscillation of level would be mainly controlled by the supply received from above.

II.—*Any reduction of the normal oscillation of level in a natural reservoir will increase the variation of flow in the outlet; and any reduction of the normal variation of flow in the outlet will increase the oscillation of level in the reservoir.*

From equation (1) it will be seen that, if the oscillation of level of a reservoir is to be diminished, $(S - Q)\,dt$ must be diminished, and consequently the values of $(S - Q)$ must be diminished; but since the variation of S from its mean is greater than that of Q, and since the variation of S is here assumed to remain unchanged, Q must be brought nearer to S, and its own variation must be increased.

The truth of the above proposition is readily apparent without mathematical demonstration. If a reservoir be prevented from rising by any amount, the water represented by the storage under normal conditions must be run out, and the flow in the outlet must be correspondingly increased. With a falling reservoir exactly the opposite process applies, and the flow of the outlet must be diminished by any restriction of the normal amount of fall. Finally, if the oscillation of level be eliminated altogether, Q will become equal to S—the condition of an unreservoired stream.

The importance of this proposition arises from its relation to the problem of controlling the oscillation of levels in the Great Lakes.

The principle laid down in Proposition II is at first sight inconsistent with equation (2). In discussing that equation it was seen that $f(h)$ increases or diminishes with h. But it has just been shown that a reduction of the fluctuations of h from its normal condition will increase the variation of Q, or $f(h)$. The explanation of this apparent paradox is, that any change in the normal variation of h[1] can only be brought about by some modification of the form or coefficients of $f(h)$; and that while h and $f(h)$ increase and decrease together for a fixed condition of the outlet, the rule has no application to a change from one condition of the outlet to another.

The converse of Proposition II is not necessarily true; that is, that an increase of oscillation in reservoir level, or of variation in outlet discharge, will diminish the variation or oscillation in the other. See remarks on artificial reservoirs under Proposition I.

III.—*In a series of reservoirs, such as has been described, if a permanent negative supply be introduced at any point, that is, if a portion of the supply be permanently withdrawn from any reservoir, it will cause a lowering of the mean level throughout the entire system below.*

This is apparent from equation (2) in which Q and h increase or diminish together. If, therefore, Q be permanently diminished, h must also be diminished, and the mean level of the reservoir will be lowered.

The lowering of the mean level in any reservoir, under the above assumption, will be such as to make $\frac{q}{c} = h'$, in which q is the negative supply and h' is the resulting fall in the mean level.

A permanent negative supply, however, unless of great relative magnitude, would have but little influence on the range of oscillation in the reservoir, provided the range of variation of S were not changed. If $f(h)$ were a right line, there would be no change in the oscillation, but if the function were of a form in which h appears at a higher power than unity, the oscillation of level would be increased by any lowering of the mean level.*

Under the assumption upon which Proposition III is based, the fall in mean level, resulting from the introduction of a negative supply, cannot be prevented by artificially increased storage in any part of the system. For assuming that $\int Q\,dt$ for the cycle is not to be changed, $\int + q\,dt$ must equal $\int - q'\,dt$, in which q and q' are the rates at which storage is accumulated during part of the cycle, and expended during the remainder. The total supply to the reservoir below is therefore not changed, and any increase during one period can be secured only by a corresponding decrease during a previous period. This might alter somewhat the oscillation of level, but would be powerless to prevent a lowering of mean level due to a permanent reduction of supply.†

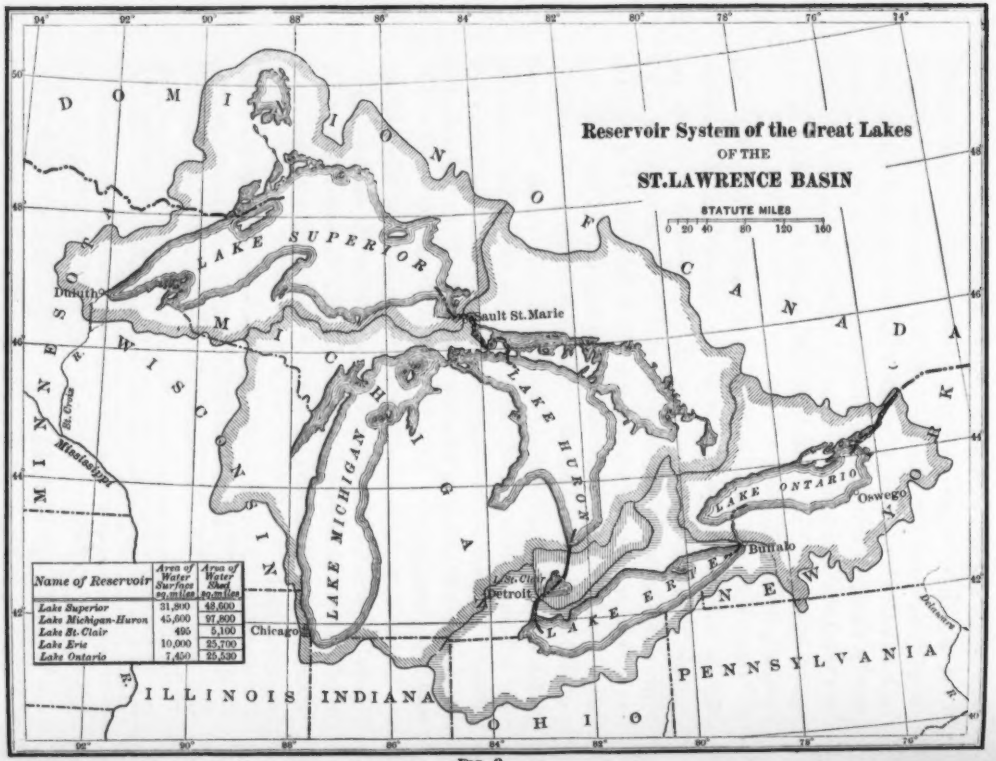
Such a result can be prevented only by changing the form or coefficients of $f(h)$ in all outlets below the point where the negative supply is introduced—in other words, by contracting the outlets so as to diminish the normal discharge at mean level by an amount equal to the negative supply.

* This may be a proper place to notice a fact which is of some interest as a theoretical refinement, although of no practical importance.

The mean level of a reservoir will rise as its fluctuations of level are diminished, and fall as the fluctuations are increased—the mean discharge remaining the same—and the mean level will stand highest when there is no fluctuation of level, or when the flow in the outlet is constant.

This law results from the nature of $f(h)$ in equation (2). If it were a right line, that is, if the variation in discharge per unit variation in h were the same at all points of h, then the mean level would always be the same for the same discharge. But $f(h)$ is nearly always a curve in which its value increases more rapidly than that of h. In the weir formulas, for example, h appears at the $\frac{3}{2}$ power, while in ordinary streams the parabolic function, ch^2, is true within narrow limits. It is obvious, therefore, that $\int f(h)\,dt$ for values of h above the mean level, must be greater than for the same values below, and since the mean discharge during the cycle cannot be increased, the plane of mean level must be somewhat below that corresponding to a constant discharge. In general, however, the difference of mean level would be inappreciable.

† Referring to the last preceding note, it will be seen that if the storage in the reservoir be so manipulated as to reduce the oscillation of levels in the reservoirs below, the mean level would be slightly raised; and if so manipulated as to increase the oscillations, the mean level would be slightly lowered. But such effects would ordinarily be infinitessimal in value.



Name of Reservoir	Area of Water Surface sq. miles	Area of Water Shed sq. miles
Lake Superior	31,800	49,600
Lake Michigan-Huron	45,600	97,800
Lake St. Clair	495	5,100
Lake Erie	10,000	25,700
Lake Ontario	7,400	25,530

FIG. 2.

IV.—*Assuming the supply to remain unchanged, no modification of the normal oscillation of level in a reservoir, or of the normal variation of flow in the outlet, is possible except by some modification of the form or coefficients of* $f(h)$.

This proposition embodies the entire question of the character of works in the outlets of reservoirs designed to control or modify the normal oscillations of h and Q. Without entering at all into details of construction, it may be broadly stated that works of this class must take one of two forms, depending upon the immediate object to be attained. If that object be a reduction of oscillation of levels in the reservoir, then the controlling works must be of such a character as to increase the value of c [equations (3), (4), (5)]. If, on the other hand, the object be to restrict the variations of flow in the outlet, then the controlling works must be of such a character as to diminish the value of c. The first purpose may be accomplished by widening the outlet and diminishing the depth, as by the use of a long-crested weir around the head of the outlet, and the second by narrowing the outlet and increasing the depth.

Inasmuch as works of improvement, such as the deepening of natural channels, and works of simple contraction, such as wing dams, piers, etc., generally increase the depth and diminish the width, such works tend to uniformity in outlet discharge at the expense of uniformity of level in the reservoir.

Application of the Foregoing Propositions to the Reservoir System of the Great Lakes.

This system embraces five distinct reservoirs, the names and areas of which, together with the areas of the tributary water-sheds, are as follows:

Name.	Area of water surface.	Area of water-shed.
Lake Superior	31 800 square miles.	48 600 square miles.
Lake Michigan-Huron*	45 600 " "	97 800 " "
Lake St. Clair†	495 " "	5 100 " "
Lake Erie	10 000 " "	25 700 " "
Lake Ontario	7 450 " "	25 530 " "

* In this discussion Lakes Michigan and Huron will be considered as a single body of water.

† The areas in this table are taken from the report of the U. S. Deep Waterways Commission for 1896, except that the water-shed of Lake St. Clair, as given in that report, has been diminished by 1 220 square miles, and this area added to that of the Lake Erie water-shed. The water-shed map of the Commission report includes a considerable area in the St. Clair water-shed which would seem to drain into the Detroit River below a point where it could have any effect upon the levels of Lake St Clair.

FIG. 2.

The connecting channels are of considerable length and irregularity, but the retardation of effects, due to the time required for the water to flow through them, is probably in no case greater than twenty-four hours. It may be much less, if those portions are omitted which are practically inlets of the lakes, taking their stage directly from that of

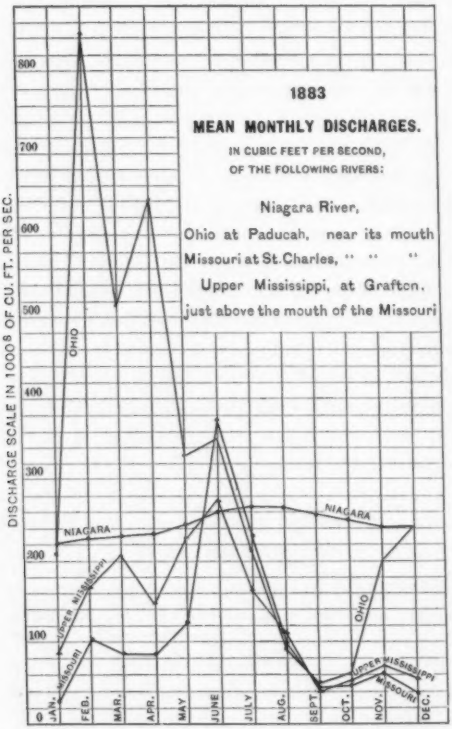


FIG. 3.

the lakes, and only those stretches are considered where the stage is dependent solely on the discharge.

Although existing data do not permit the precise determination of the values of c for the various outlets, they may be stated within a probable limit of error that will answer all the purposes of a rigid demonstration.

The values of c, here assumed, are as follows:

For Sault Ste. Marie River.....17 000 second-feet.
" St. Clair and Detroit Rivers.....26 000 " "
" Niagara River.....30 000 " "
" St. Lawrence River..... 34 000 " "

Deductions from Proposition I.

To appreciate fully the moderating effect of these reservoirs upon the stream flowing through them, as explained under Proposition I (page 359), it will be of interest to make a comparison with other well-known streams of similar magnitude. Take, for this purpose, the Ohio River at its mouth, the Missouri at its mouth, and the Mississippi at the mouth of the Missouri, and let them be compared with the Niagara at Buffalo. The areas of water-sheds of these four streams, and their mean discharge in second-feet, are as follows:*

	Niagara.	Ohio.	Missouri.	Mississippi.
Water-shed in square miles.....	265 095	205 750	530 810	171 570
Discharge in second-feet.....	232 800	307 000	100 000	130 000

The accompanying diagram, Fig. 3, exhibits graphically the variations in mean monthly discharge of these streams for the year 1883. The actual maxima and minima show, of course, a considerably larger divergence than the monthly means, the ratios $\frac{\text{(maximum discharge)}}{\text{(minimum discharge)}}$ for the above year being, for the Ohio, 28.22; for the Missouri, 29.00, and for the Mississippi, 10.29. For Niagara the ratio, disregarding wind effects, probably does not exceed, in any one year, 1.50.

In terms of absolute quantities, the storage by which this result is produced may be readily determined. The mean annual oscillation of level in Lake Superior, based upon 25 years' observations, is 0.93 ft., in Lake Michigan-Huron, 1 ft., and in Lake Erie, 1.16 ft. These oscillations represent an aggregate storage of 2 419 billion cubic feet of water—greater than the over-flow excess of the Mississippi River at Cairo in the great flood of 1897, and equivalent to about 153 000 second-feet for a period of six months. The maximum annual oscillation is about twice the above mean, and, of course, represents twice the amount of storage.

* The above data for discharge are based upon 25 years' record (1871–1895) for Niagara, and 6 years' record (1880–1885) for the other streams.

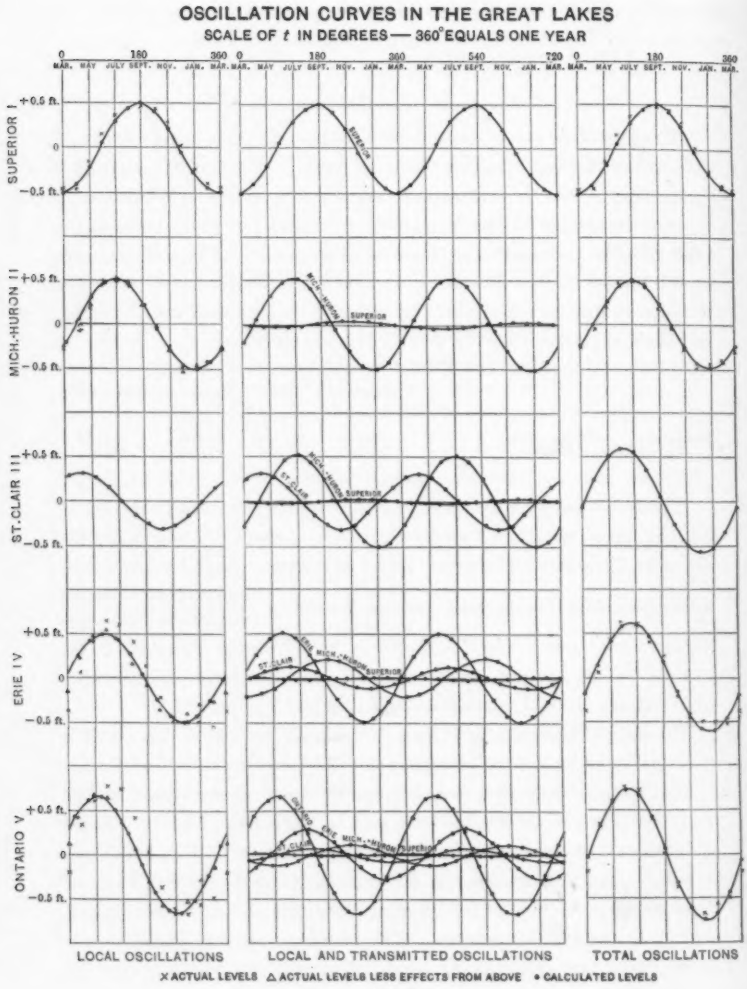

OSCILLATION CURVES IN THE GREAT LAKES
SCALE OF t IN DEGREES — 360° EQUALS ONE YEAR
0 180 360 0 180 360 540 720 0 180 360
MAR. MAY JULY SEPT. NOV. JAN. MAR. MAR. MAY JULY SEPT. NOV. JAN. MAR. MAY JULY SEPT. NOV. JAN. MAR. MAR. MAY JULY SEPT. NOV. JAN. MAR.
SUPERIOR I
MICH.-HURON II
ST. CLAIR III
ERIE IV
ONTARIO V
+0.5 ft.
0
−0.5 ft.
LOCAL OSCILLATIONS
LOCAL AND TRANSMITTED OSCILLATIONS
TOTAL OSCILLATIONS
× ACTUAL LEVELS △ ACTUAL LEVELS LESS EFFECTS FROM ABOVE • CALCULATED LEVELS

FIG. 4.

Not less in magnitude and importance is the cyclic oscillation referred to earlier in this paper. During the period from 1872 to 1876, for example, the mean annual levels of the lakes above Niagara rose continuously, the aggregate storage being about 4 000 billion cubic feet. During the next three years the levels fell continuously, the aggregate loss of storage amounting to about 3 600 billion cubic feet.

The enormous moderating influence of these vast storage reservoirs thus practically eliminates, not only the regularly recurring changes in supply, but all sudden, erratic, or transient irregularities, and produces a flow in the outlets which, as already stated, is practically uniform from one year's end to the other.

In applying the formulas (3), (4) and (5), to the Great Lakes, no attempt has been made to go farther than to adapt them to the mean curves based upon 25 years' observations. To these, the ordinary sine curve, used in the analysis, has been fitted as closely as possible. The correspondence of the curves is not exact, but is near enough not to affect the general conclusions drawn from the discussion.

The following table gives the value of the retardation R, and the coefficients of reduction P and p, for the five lakes:

Name of lake.	R.	P.	p.
Superior	85.7	0.096	.....
Michigan-Huron	85.3	0.102	0.067
St. Clair	6.1	0.994	0.994
Erie	62.5	0.475	0.412
Ontario	51.3	0.635	0.560

This table, and the accompanying oscillation curves, Fig. 4, form a complete exhibit of the local and transmitted elements of the aggregate oscillation in each unit of the system.*

Column (2) explains the well known fact that the highest and lowest stages in the lakes occur long after the periods of maximum and minimum of supply have passed. However, from the curves and from column (2) the approximate mean dates of maximum or minimum supply for each of the lakes can be determined. To do this, take from the curve of oscillation due to local supply in any lake the dates of its maximum and minimum, and from these subtract the corresponding retardations in column (2). The results deduced by this process are

*The effect of the variable supply to Superior consumes 291 days in reaching Ontario, and its maximum is then only 1% of the total maximum oscillation in the latter lake.

given in the table below, and fall well within the probable limits of actual occurrence.

Name of Lake.	Date of actual h max.	Date of local h max.	Date of actual h min.	Date of local h min.	Retardation.	Date of S max.	Date of S min.
Superior	Sept.	Sept.	Mar.	Mar.	86	June	Dec.
Michigan-Huron	July	July	Dec.	Jan.	85	April	Oct.
St. Clair	July	April	Dec.	Oct.	6	April	Oct.
Erie	June	June	Feb.	Dec.	63	April	Oct.
Ontario	June	May	Dec.	Nov.	51	April	Sept.

The quantity S, it must be borne in mind, is a very complex one. It is the result of at least three important and ever active causes—precipitation upon, and evaporation from, the surface of the reservoir, and run-off from the tributary water-shed. The first and third elements are always positive and the second always negative; but their maxima and minima are rarely coincident, and probably none of them coincide very closely with the actual aggregate. Precipitation, as based upon many years' observations, has two yearly maxima, occurring usually in June and September, and two minima, occurring usually in August and February. Run-off has its maximum generally in those months when the melting of the winter snows occurs. This is ordinarily in March for the lower lake region, and in April for the Lake Superior region. The minimum of run-off spreads over a pretty long period, ranging from September to February, and is controlled, not only by actual precipitation, but by the influence of cold weather in congealing precipitation and checking the flow of streams. The maximum of evaporation occurs generally in July and August, and the minimum in the period from December to February; but there are certain phenomena, to be referred to later on, which indicate a higher winter rate of evaporation than is generally supposed.

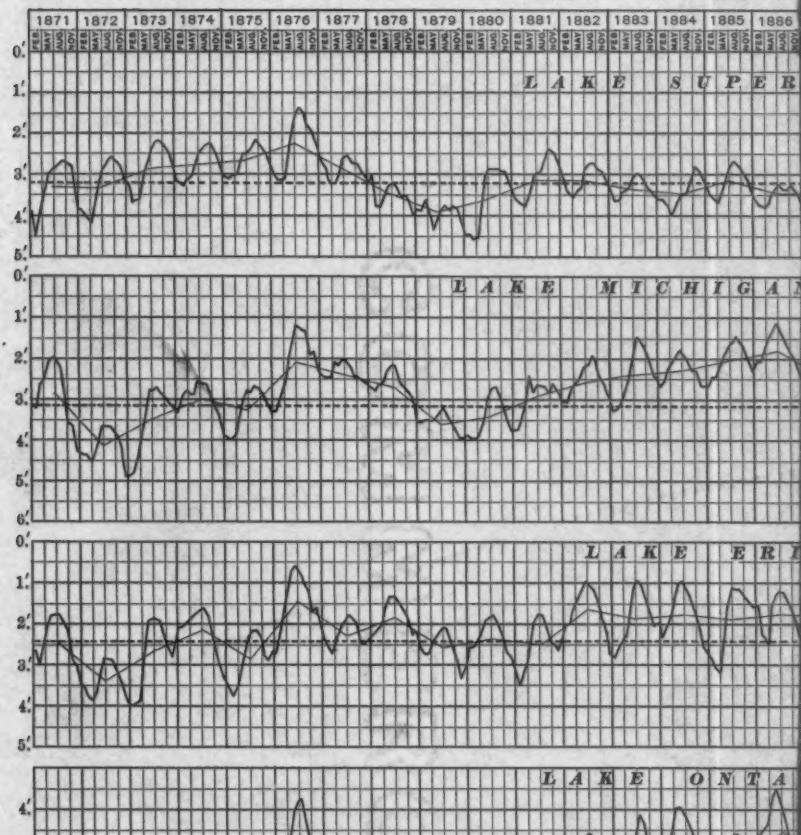
The complex and variable elements, of which the actual local supply for any of the lakes is made up, render it extremely hazardous to predict from them the time of actual maximum and minimum; for these may incline toward any of the component elements which happen at the time to predominate. No method seems so rational as that above applied of working backward from the known dates of maximum and minimum oscillations due to local supply.

Assuming the mean discharge of the outlets to be, for Superior, 75 000 second-feet; for Michigan-Huron, 195 000 second-feet; for St.

CHART SHOWING THE OSCILLATIONS OF LEVEL AND C

GREAT LAKES OF THE ST. LAW

FROM 1871 TO 1897 INCLUSIVE, WITH MEANS OF TWENTY-FIVE



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M 1871 TO 1897 INCLUSIVE, WITH MEANS OF TWENTY-FIVE YEARS FROM 1

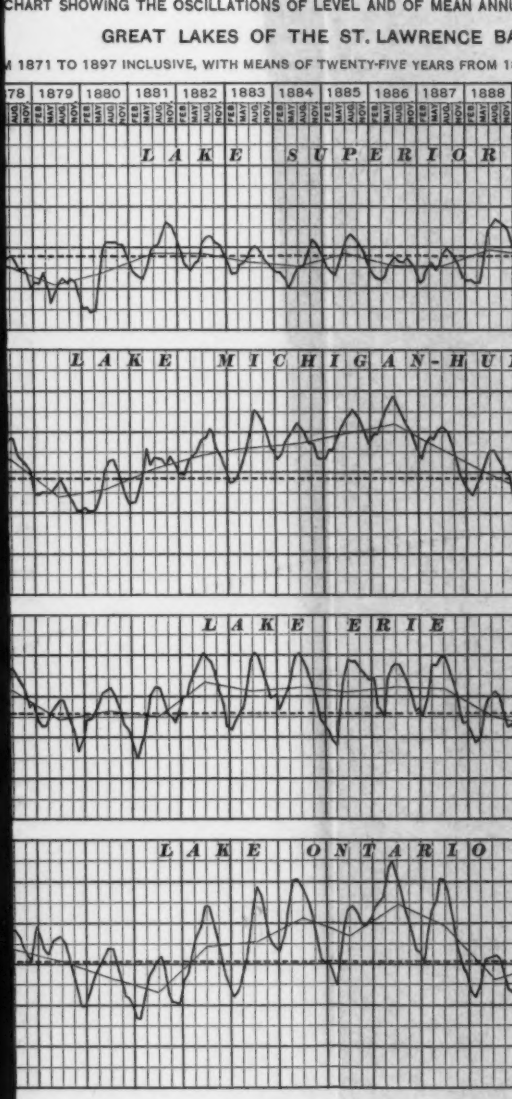

PLATE XXXVI.
TRANS. AM. SOC. CIV. ENGRS.
VOL. XL, No. 839.
CHITTENDEN AND SEDDON
ON LEVELS OF THE GREAT LAKES.

MEAN ANNUAL LEVEL OF THE

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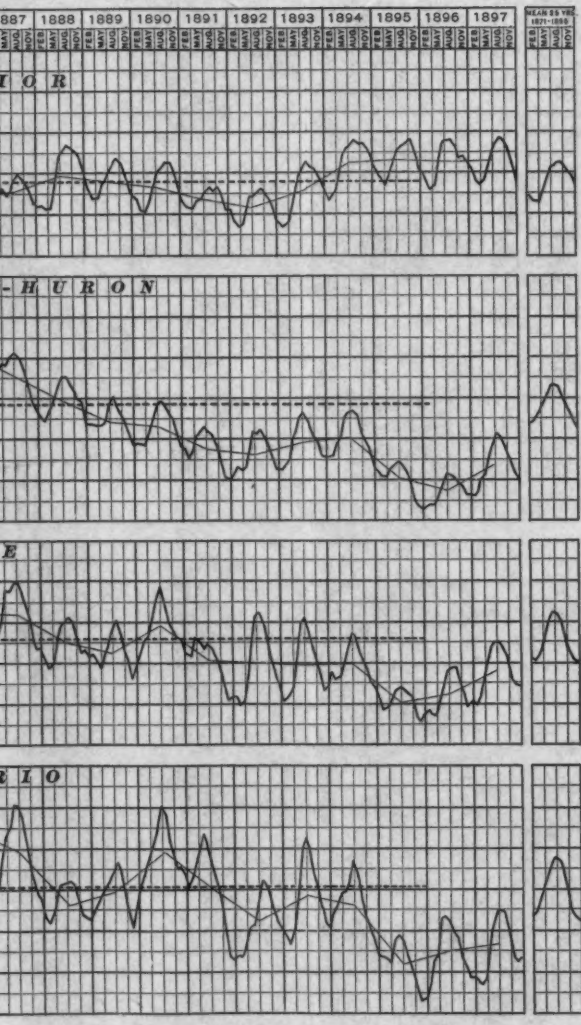
ARS FROM 1871 TO 1895 INCLUSIVE.



Clair, 200 000 second-feet; for Erie, 225 000 second-feet; for Ontario, 260 000 second-feet; the corresponding local supply for each of the lakes in thousands of second-feet per month, based upon the twenty-five year curves, is found to be as follows:

Month.	Superior.	Michigan-Huron.	St. Clair.	Erie.	Ontario.
January	— 2	+111	+19	+43	+62
February	+ 31	+185	+21	+52	+74
March	+ 75	+235	+20	+55	+75
April	+115	+255	+15	+49	+66
May	+152	+240	+ 8	+37	+49
June	+164	+191	+ 1	+23	+27
July	+153	+129	— 5	+ 7	+ 8
August	+119	+ 55	— 7	+ 2	— 4
September	+ 75	+ 5	— 6	— 5	— 5
October	+ 31	— 15	— 1	+ 1	+ 4
November	— 2	+ 0	+ 6	+13	+21
December	— 14	+ 47	+13	+27	+43

The results here given are to be taken only as general approximations, in the absence of precise data as to the mean discharge of the outlets; but they serve to show the fact, intended to be brought out here, that at certain seasons there is a negative supply on all of the lakes. The cause will naturally be attributed to the preponderance of evaporation at such times over rainfall and run-off; but the perplexing feature of the phenomenon is that in one lake in particular—Superior—this negative supply comes in months when evaporation is supposed to be at a minimum. An examination of Table No. 2 will show that on all the lakes there are large deficiencies at times, even in winter months. No explanation suggests itself except that evaporation from ice and snow on the lakes may be greater than is commonly supposed.

Relative Effects of Local and Transmitted Supplies.—Plate XXXVI, showing oscillation curves, affords a striking illustration of the observation made in discussing Proposition I (page 359), viz., that, in the general case, the oscillation of levels will be controlled by the transmitted supply when the reservoir is relatively small, and by the local supply when it is relatively large. Thus, the maximum of oscillation in Michigan-Huron, if due to local supply alone, would be 4% greater than it actually is, and would occur about three days earlier. The maximum of oscillation, if due to the transmitted supply from Superior alone, would be only 7% as great as it actually is, and would occur 141 days later.

For St. Clair, the local maximum would be about 53% as great as the actual, and would occur about 57 days earlier; while the trans-

mitted maximum would be 83% as great as the actual, and would occur 32 days later.

For Erie, the local maximum would be about 80% as great as the actual, and would occur about 20 days earlier; while the transmitted maximum would be about 39% as great as the actual, and would occur 51 days later.

For Ontario, the local maximum would be about 88% as great as the actual, and would occur about 28 days earlier; while the transmitted maximum would be about 47% as great as the actual, and would occur about 64 days later.

From these figures, and from the curves, an excellent idea may be had of the parts which local and transmitted supplies play in the actual oscillations of the lakes. Except in St. Clair, it is seen that the local supply is the controlling factor, and that in one lake—Michigan-Huron—the transmitted supply actually diminishes the total fluctuation. In St. Clair the transmitted supply controls—as was to have been expected on account of its relatively small area,

It is also plain that any attempt to control the fluctuations of one lake by modifying the inflow from another will produce only insignificant results.

In like manner it is manifest that the oft-asserted mutual dependence of levels between Michigan-Huron and Erie, evidence of which, it is urged by some, is the fact that their difference of level scarcely ever varies as much as 1 ft.,* has no foundation in fact. St. Clair completely interrupts the continuity of flow on account of the fluctuation due to its own local supply.†

* Lake Michigan-Huron and Lake Erie rise and fall together, not because of any mutual dependence of levels, but because similar climatic forces are operating simultaneously in both basins. The fact that their levels do not move in closer unison—that sometimes Erie is falling when Michigan-Huron is rising, and *vice versa*—still further negatives the probability of any very intimate relation between the oscillations of level in these two bodies of water.

† The influence of St. Clair upon the discharge of the Detroit River may even be greater than has been here assumed. Unfortunately there are no actual data as to the oscillation of levels in this lake, and it has been necessary to deduce a curve from known data, such as the area of the lake, the inflow from Michigan-Huron, and an assumed S, based upon the similarity of climatic conditions on the watersheds of Michigan-Huron, Erie and St. Clair. In this, however, an error may have been made in assuming an eccentricity for the St. Clair curve of supply but little greater than that for the other lakes. The water-shed of St. Clair, compared with the lake area, is relatively much larger than those of the other lakes, as may be seen from the following ratios of lake to land area:

Superior	1.5
Michigan-Huron	2.1
St. Clair	10.3
Erie	2.4
Ontario	3.4

Remembering that S is the algebraic sum of precipitation on the lake surface, evaporation from it, and run-off from the water-shed, a little consideration of the character of these elements will show that as run-off predominates over rainfall and evaporation, the eccentricity of S will increase. It is quite possible, therefore, that the range of local supply for St. Clair has not been assumed large enough, and that the oscillations of level in that reservoir are due, to a greater extent than has been admitted, to local supply.

Deductions from Proposition II.

Table No. 2, appended to this paper, shows in detail, month by month for twenty-five years, and for each of the five lakes, the amount of storage accumulated or lost, both in terms of h and in thousands of second-feet per month. It also shows similar data for the mean of the twenty-five years. Table No. 3 shows the oscillation of mean annual level in feet in each of the lakes from 1871 to 1895, inclusive, with the equivalent storage in thousands of cubic feet for the period of one year. Table No. 4 shows the rate in second-feet at which storage corresponding to a rise or fall of 6 ins. on each of the lakes would have to be accumulated or expended in periods varying from one to three months. The tables show, not only the data for each lake by itself, but the cumulative effects in descending the series, supposing the causes to be operating simultaneously throughout the system.

These tables supply ample data for conclusions as to the practicability of controlling the levels of the lakes within fixed limits—say, 6 ins. Thus, let it be assumed that the outlet of any of the lakes, Superior, for instance, is to be so regulated that the annual oscillation of the lake level shall not exceed 0.5 ft. The amount of storage represented by a depth of 0.5 ft. on Lake Superior is equivalent to 168 000 second-feet for one month. Now, during the months from March to September, in 1871, for example, storage was accumulated in this lake at the following rates in thousands of second-feet: March, 146; April, 168; May, 178; June, 40; July, 24; August, 20; September, 34. Assuming that this storage is to be reduced to an equivalent of 168 000 second-feet for one month, and that the reduction is to be taken from the months of most rapid accumulation, it would give the following results: It would reduce the storage in March by 38, in April by 60, and in May by 70, and would leave for each of these months 108 000 second-feet* which would have to be run out in addition to the normal discharge. This, it will be understood, is based upon the impossible supposition that climatic conditions can be foreseen so as to apply the allowable storage of 0.5 ft. to those months of most rapid accumulation. In actual practice, of course, no such result could be realized, and the increase of outflow would be greater than above indicated.

Applying this process to the ensuing period of falling levels, from October, 1871, to April, 1872, it will be found that for the months of

* For values of mean discharge of the various lakes, see p. 375.

December and January the outflow would have had to be reduced by 94 000 second-feet *, and for the other months, during which the lake was falling, by the full amounts given in the table. It requires no comment to show, not only the impracticability, but the utter impossibility of any such accomplishment.

However, the difficulty here encountered would evidently be less in the lower units of the system, if considered by themselves, for there the reservoir area is much less and the flow of the outlets much greater. To see what effect would be produced upon the flow of Niagara by a reduction to 0.5 ft. of the oscillation in Erie, apply the above process to that lake for the year 1871–72. It will be found that the outflow would not have been increased by more than 24 000 second-feet, nor restricted by more than 32 000 second-feet. There are other years, however, in which the increase of outflow would have been much greater, but it may be assumed that any practicable restriction of the oscillation of Erie to a limit of 0.5 ft. would probably not often increase or decrease the flow in the outlet by more than 30 000 second-feet.* Whether a change of this amount in the normal variation of flow of Niagara, particularly in the low-water period, could be admitted, is at least an open question, and one that would have to be settled before regulation could actually take place.

If regulation to a limit of 6 ins. were simultaneously applied to all the lakes, or even to Michigan-Huron and Erie, the cumulative effect on Niagara would render the project wholly impossible.

It will probably impress any one who carefully examines this subject that an interference with the normal annual oscillation of levels of the lakes will at best be an uncertain and hazardous undertaking; and that, if such interference were to take place, it might better be for the purpose of increasing, rather than diminishing, the oscillations of levels in the reservoir, and of thus reducing the variations of flow in the outlet.

It is, after all, not so much the annual oscillation that gives rise to complaint on the part of navigators as the long periodic oscillation. The real source of trouble is the continuous subsidence of levels through several years, by which the carrying power of the great ships is constantly reduced, and their earning capacity diminished. Can these periodic oscillations be eliminated, and can the mean annual levels be always maintained above a minimum plane?

* For values of mean discharge of the various lakes, see p. 375.

So far as the rise is concerned, the problem is certainly feasible, for the increased flow, due to its elimination, when spread over such long periods, would not be objectionable, even in its cumulative effects.

The case is not equally clear in regard to the subsidence of levels. An extreme example of the fall of the mean annual levels of Michigan-Huron and Erie is that of the year 1894–95. Michigan-Huron dropped during that period at a rate of 42 000 second-feet, and Erie at the rate of 8 220 second-feet; but Superior in the meantime rose by an average of 1 700 second-feet. To have prevented the subsidence of levels in Michigan-Huron and Erie would have required a uniform reduction of the normal discharge of Michigan-Huron by 40 300 second-feet, and of Erie alone by 8 220 second-feet, while the cumulative effect in Erie would have been 48 520 second-feet. This being a period when the normal discharge of these rivers was injuriously small, these reductions would probably not have been admissible during the navigation season. If restricted to one-third of a year, that is, to the winter season, they would have been 120 900 second-feet for Michigan-Huron, 24 663 second-feet for Erie alone, or 145 560 second-feet as the cumulative effect from Michigan-Huron and Erie. As the normal discharge of Michigan-Huron at this time was probably between 150 000 and 175 000 second-feet, and of Erie not much more than 200 000 second-feet, the elimination of the cyclic fluctuation alone would, it is seen, have involved a reduction of discharge in the outlets, during the entire winter season of four months, of nearly 75 per cent.

Should full investigation show that this amount of reduction in the discharge of the outlets during the non-navigation season could be permitted without injury to any interests, the possibility of uniform regulation, such as is here assumed, would still be dependent upon a condition which can never be realized. It is impossible to forecast climatic conditions for anything like the periods covered by these cycles. Those charged with the control of the outlets must always be for the most part in the dark as to what is going to happen in regard to rainfall and run-off. It is therefore clearly impossible to fix in advance, even for the period of a single year, a uniform rate at which the discharge of the outlets must be increased or diminished in order to prevent a change in the mean annual level. The only way to accomplish this result that suggests itself would be to accumulate an amount of storage in the lakes during each winter, and the ensuing

period of high water, equal to the maximum subsidence which experience shows may be expected to follow during the low-water season. If this were done, the levels could always be maintained above a minimum plane. This result, however, could be attained only at the cost of increasing now and then the annual oscillation.

Of course, to make any regulation of the lake levels practicable the mean levels must be raised somewhat, in order to be able to secure the increase of outflow which would at times be necessary; but this elevation of mean level is in itself one of the ends contemplated by the general scheme of lake level regulation. Taken by itself it is unquestionably a practicable matter, at least within small limits. Such a step would indeed involve grave difficulties in the outlets themselves, to which reference will presently be made, and will always be complicated with the question of damages, but so far as the simple matter of raising the levels of the lakes is concerned, there can be not the slightest doubt of its feasibility. It is in fact claimed that this has already been accomplished in Lake Superior by works placed in the outlet of St. Mary's River for other purposes. These works consist of bridge piers and water-power works, and the contraction of the outlet has been such as to give an estimated rise of 0.5 ft. in the level of the lake, prior to January, 1896.*

The question of permanently raising the levels of the lakes is, or may be made, entirely independent of that lake level regulation. A simple contraction of the outlets will accomplish the first purpose, but not the second. It would in fact increase the oscillation somewhat, in accordance with a principle brought out in discussing Proposition IV (page 369), that if the outlet of a reservoir be narrowed or deepened, the value of c will be diminished, and the oscillation of level will be increased while the variation in flow will be lessened.

Deductions from Proposition III.

The third of these general propositions relates to the effects upon the lake levels, of the diversion of any portion of the supply of the

* Report by E. E. Haskell, U. S. Asst. Engineer, dated January 11th, 1896. Although it is true that the level of Lake Superior was rising for several years prior to 1896, while the levels of the lower lakes were falling, still the climatic conditions are dissimilar enough to lead one to hesitate in attributing this particular rise to artificial causes alone.

Great Lakes to other watersheds, and to the methods of counteracting such effect. The specific application of the proposition now in view, is to the Chicago Drainage Canal with its contemplated diversion of 10 000 second-feet, and to the scheme advocated by some engineers of diverting a much larger amount, say 30 000 second-feet, to help out the low-water navigation of the Mississippi. That such diversions will permanently lower the levels of all the lakes, below and including that from which they are taken, is certain. The amount of this lowering is a function of c of the various outlets, and its correct determination depends upon the prior correct determination of this coefficient.

Among the suggestions advanced for counteracting this lowering effect is that of storing water in Lake Superior to make up the amount of diversion. A reference to the discussion of Proposition III (page 366) will show the fallacy of this scheme. The storage of water in Lake Superior must commence by cutting off the outflow. When the stored water is run out, the total increase of flow over the normal condition will only be equal to the previous decrease. In other words, the total supply to the lakes below, upon which their mean level depends, cannot be altered a particle by storage.

The only way the effects of diversions can be counteracted is to contract the outlets, so that the flow through them at the normal mean level shall be diminished by an amount equal to the diversion.

Analogous to the effect which must result from the permanent withdrawal of any portion of the supply of the lake system to other water-sheds is that due to an increase of the cross-sections of the outlets resulting from improvements in the interests of navigation. Such works must necessarily result in a permanent lowering of mean level in the reservoir above. The remedy is, of course, to contract the outlets sufficiently to diminish the outflow at the normal mean level by an amount equal to the increase caused by the enlarge ment of section. In superficial area the contraction would have to be greater than the previous enlargement, for the whole change thus made in the cross-section is in the direction of deepening and narrowing the outlet; and a deep and narrow channel will carry more water than a wide and shallow one of the same area of cross-section.

Deductions from Proposition IV.

All works intended to change the levels of the lakes, to control their oscillations, to counteract diversions, or to regulate the flow of the outlets, must be placed directly in, or at the head of, the outlets themselves. The two broad classes into which such works must fall have already been pointed out, viz.:

(*1*) Works tending to increase the width and diminish the depth, with the result of diminishing oscillation of level and increasing variations of discharge; and

(*2*) Works tending to contract the width and increase the depth, with the result of increasing oscillation of levels and diminishing variations in discharge.

The only example of a work under the first class is that proposed by some engineers for controlling the level of Lake Erie. For obvious reasons, the regulation of the levels of this lake is of more importance than that of any of the others. There need only be mentioned its vast harbor interests, and the fact that at least two, and probably three, canals will in the future lead directly from its lower extremity to Ontario or to the Hudson River. A fixed depth in these harbors and over the miter sills is of great importance. To secure uniformity of level it has been proposed to enclose the head of Niagara by a fixed weir, which will give the desired permanent increase in elevation of level, and which shall be of such length that a change of depth on the crest of, say, 6 ins. will so modify the outflow as to prevent a rise or fall of level of more than that amount.

Two objections appear to be conclusive against this arrangement: (*a*) The very advantage which first commends it, that of automatic regulation of oscillations, would, in reality, prove a serious disadvantage; and (*b*) the whole result can be accomplished in a simpler, more effective, and less expensive way by works placed directly in the outlet.

The trouble with the automatic action of the long-crested weir is that it would always give the same discharge for the same depth of overflow; whereas, the best regulation of the discharge might require that it be not always the same for the same stage. It has been seen that a regulation of the levels of Erie within a limit of 6 ins. would increase or decrease the normal flow in Niagara at times by as much as 30 000

second-feet; but the range of the normal variation itself above or below the mean doubtless frequently amounts to 30 000 second-feet. The weir must therefore be of such length that a variation of 6 ins. in the depth of flow over its crest will give a variation of discharge of 60 000 second-feet. When it is considered that changes of 2 to 4 ft. in gauge readings due to wind effects "are of so frequent occurrence as to be considered common,"* and that changes of from 4 to 8 ft. are not rare, the effect of a long weir upon the discharge of Niagara, in times of heavy winds, would, it is readily seen, be disastrous. With a rising lake the matter would be of less consequence; for if the level of the lake were regulated but a foot or two above its present mean stage, the increased overflow would choke up the outlet and drown out the weir; but with a falling lake, no such counteracting effect would be experienced, and Niagara River would frequently run dry.

As before stated, the whole result expected from the long weir can be obtained in a simpler, more effective and less expensive way, by works placed directly in the outlet. The form of structure which appears to be essential to a rational regulation of the levels of the lakes is one that shall be in a measure under the control of human agencies. Disclaiming any purpose of proposing technical or specific details of construction, the following general plan may be suggested as embodying what would seem to be the most practicable method of control. At a suitable section of the outlet, preferably where rock foundation may be had, let a series of piers, similar to bridge piers, be erected at proper intervals, whose aggregate area of cross-section below the water surface will be equal to the contraction of the channel section, which is necessary to secure the desired permanent elevation of the mean level of the lake. These piers would, thus, by themselves, accomplish that part of the purpose of regulation which relates to a permanent elevation of level. For the purpose of controlling the oscillations, some arrangement by which the area of the new cross-section can be enlarged or diminished is necessary. To accomplish this, let a sill be anchored to the bottom of the river from pier to pier, and let a bridge be laid from pier to pier, but slightly elevated above the water surface. Upon this bridge, at convenient intervals, let proper arrangements be made for the handling of good-sized needles

* Report of the United States Deep Waterways Commission, for 1896, p. 156.

which may be lowered into the river and supported by the bridge at the upper end, and by the sill at the lower end. Or, better still, these needles could be fixed permanently in position, but balanced on the longitudinal axis, so that the broad flat surface or the narrow edge, as desired, could be presented to the current. These needles, when in place, need not be adjacent, but there may be open intervals between them; the main point being that their aggregate area shall be sufficient, by proper manipulation, to give the desired control over the flow in the outlet. A navigable pass of sufficient width should be left entirely free from obstruction. With an arrangement of this sort, all the advantages of the fixed weir may be realized, with the very important additional advantage of adaptation of control to actual conditions as they arise. Such a structure ought to be much cheaper than a long weir. Its care and management would cost something, but the expense from this cause ought not to be great.

One important drawback will always be encountered with any form of contraction that can be devised. The raising of the level of the lakes will increase the slope at the site of the works of contraction, and may develop currents of such rapidity as to interfere seriously with the passage of boats. At the Sault Ste. Marie this is a matter of less importance, for locks there are a necessity any way, and the only effect would be to increase the lift. Even this might not result if Michigan-Huron and Superior were each raised by a like amount. But in passing from Huron, St. Clair or Erie into the rivers below, the case is different, and contracting works might, and probably would, develop strong currents and sharp slopes in their immediate vicinity.* It is doubtful if the simultaneous and equal raising of Erie and Michigan-Huron would obviate the difficulty, for the increased channel depth would diminish the slopes, and would tend to concentrate the fall at the contracting works. It might be necessary to have a series of works, one below another, with the area of contraction gradually diminishing so as to let the slope down gently by distributing it over a considerable distance. In any event, the use of locks would be too serious an inconvenience to merit consideration.†

* The outlets of both Michigan-Huron and Erie have engorged sections where they leave the lakes; and even in their natural conditions show marked slopes and high velocity of flow at these points.

† It is scarcely necessary to invite attention to the international feature of this problem. No radical step in the direction of lake-level regulation can be taken, except with the co-operation, or at least consent, of Canada. It is, in fact, by no means improbable that this feature of the question may prove the most embarrassing of all.

Physical Data.

It will have been observed that, in this discussion, the primary data upon which the oscillation of lake levels depends—those, namely, of precipitation, evaporation, and run-off—have been dealt with scarcely at all. In fact S does not appear in equations (3), (4) and (5), but only the quantities A, h and c. The discussion of the reservoir question therefore depends upon a knowledge of the area of the lakes, the oscillation of the levels, and the characteristics of flow in the outlets. The first of these elements is known with precision. The second has been a matter of record for nearly forty years, and in this discussion is based upon a period of observation of twenty-five years. The data are not, indeed, all that could be wished. The records have not been made up from self-registering gauges, as would have been the better plan, but from observations taken at a particular hour each day. Considering the sudden effects of winds and barometric changes on the levels of the lakes, it is clear that the individual records cannot be of much value; but since erroneous values are liable to occur as much on one side of the truth as on the other, the monthly means of daily readings are probably not far from correct, while the mean of so long a period as twenty-five years must be very accurate.

Of the discharge of the outlets comparatively little is yet known. In this matter, however, it is not so much the absolute value of the mean discharge that is needed, as the variations from the mean. In other words, Q is of less importance than c. In the assumption of c it has been necessary to proceed somewhat in the dark, but nevertheless with confidence, because a considerable variation in c will not materially affect the results with reservoirs of such magnitude as the Great Lakes. In like manner the assumption that $f(h)$ is a right line is so near the truth that its departure therefrom cuts no figure whatever in the general result. It is confidently believed, therefore, that the general conclusions here arrived at will not be essentially modified by more exhaustive data.

It is, nevertheless, of great importance to a final solution of the problems of the lakes that these data be made as complete as possible. The future records of the lake levels should be kept with self-registering gauges, and there should be one of these for Lake St. Clair as well

as for the other lakes. The discharge measurements should embrace every phase of flow in the outlets, and particular care should be exercised to secure simultaneous measurements at various stations along the channels connecting Huron with Erie. It is a matter of congratulation to know that these important steps are now in contemplation by the existing Deep Waterways Commission.

Concerning S, the collection of data appears to be altogether of secondary importance, and this for the excellent reason, that, having A, h, c and Q, S can be determined from them more accurately than in any other way. If the difficulties in the way of a satisfactory determination of any one of the elements of S—precipitation, evaporation, or run-off—are considered, it will be conceded that an approach to the problem from that direction is wholly injudicious. Take Lake Superior, for example. The precipitation at any one of the points, Duluth, Marquette, Sault Ste. Marie, White River or Port Arthur, around the shore of the lake, varies materially from that at any other point. Admitting that correct records can be kept at these points, how is the true proportion of lake area to be assigned to each in order to determine the average precipitation over the whole?

The question of evaporation is still more unsatisfactory. There is no practicable method of determining it accurately. How are the effects of winds on such great bodies of water to be determined? It is known that waves increase the exposed area about 2½%, while the spray from breakers adds still more; but at the same time the commotion of the water brings up the cooler depths to the surface and lowers the temperature upon which the evaporation largely depends.*

Again, observations taken near shore might not hold for the interior of the lake, where the atmosphere may reasonably be assumed to be more laden with moisture and to be of a lower temperature. The element of evaporation from ice and snow would be a very uncertain one, owing to the absence of any definite knowledge of the extent of ice-covered areas. With these inherent uncertainties in the problem of determining the rate of evaporation on the lakes, great confidence can hardly attach to any attainable results.

* In midsummer the surface temperature on Lake Superior was found by Geo. Y. Wisner, M. Am. Soc. C. E., to fall 5 or 6° after heavy winds.

The run-off from the land area of the basin could be determined with tolerable accuracy for any one year, by a continuous gauging of a sufficient number of streams, but the results would be of little value for another year in which conditions might be very different.

It is therefore not apparent why this particular line of data should receive special attention. The complex expression which gives their combined effect is already integrated in the rise and fall of the lake levels and in the flow of the outlets, and if these are known, S can be determined from them better than it can ever be measured.

TABLE No. 1—Water Levels of the Great Lakes.

[From the Report of the United States Deep Waterways Commission, 1896.]

Lake Superior, at Superior, Wis., and Marquette, Mich.

Monthly mean of water levels below the plane of reference of United States Lake Survey (high water of 1838).

Elevation of the plane of reference above mean tide at New York City is 604.76 ft.

Year.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Mean
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet	Feet.
1871...	3.85	4.45	4.08	3.53	3.00	2.88	2.81	2.75	2.65	2.72	2.79	3.58	3.25
1872...	3.85	3.96	4.08	4.18	3.53	3.15	2.88	2.71	2.55	2.65	2.80	3.10	3.29
1873...	3.20	3.65	3.62	3.61	3.01	2.71	2.42	2.24	2.18	2.28	2.42	2.72	2.84
1874...	3.17	3.19	3.23	3.13	3.06	2.86	2.48	2.39	2.29	2.23	2.41	2.72	2.76
1875...	3.04	3.08	3.04	3.04	2.82	2.46	2.47	2.38	2.15	2.30	2.44	2.64	2.65
1876...	2.84	3.05	3.14	3.11	2.57	1.89	1.50	1.39	1.50	1.83	1.99	2.27	2.26
1877...	2.63	2.87	3.13	3.21	3.22	3.00	2.62	2.56	2.72	2.72	2.93	3.00	2.88
1878...	3.12	3.00	3.77	3.80	3.53	3.25	3.18	3.30	3.47	3.40	3.60	3.92	3.45
1879...	3.82	3.86	3.56	3.95	4.31	4.08	3.84	3.72	3.83	3.74	3.82	4.12	3.89
1880...	4.44	4.48	4.57	4.54	3.85	3.02	2.87	2.88	2.88	2.93	2.99	3.25	3.56
1881...	3.51	3.61	3.70	3.79	3.49	3.05	2.99	2.94	2.71	2.37	2.44	2.72	3.11
1882...	3.07	3.32	3.43	3.51	3.35	3.33	2.88	2.76	2.72	2.89	2.91	3.10	3.11
1883...	3.33	3.62	3.62	3.37	3.36	3.26	3.01	2.99	3.03	3.23	3.38	3.49	3.31
1884...	3.56	3.59	3.71	4.00	3.78	3.58	3.44	3.43	3.16	2.80	2.90	3.11	3.42
1885...	3.34	3.52	3.60	3.65	3.32	3.04	2.80	2.68	2.75	2.92	3.07	3.40	3.17
1886...	3.60	3.73	3.79	3.70	3.45	3.31	3.24	3.33	3.35	3.25	3.40	3.54	3.47
1887...	3.85	3.83	3.52	3.35	3.56	3.40	3.12	3.04	3.18	3.25	3.49	3.71	3.44
1888...	3.82	3.81	3.88	3.88	3.41	2.63	2.44	2.30	2.35	2.44	2.58	2.98	3.04
1889...	3.25	3.47	3.64	3.63	3.28	3.16	2.97	2.78	2.65	2.81	3.12	3.42	3.18
1890...	3.56	3.69	3.93	3.96	3.75	3.30	3.00	2.85	2.72	2.75	2.96	3.32	3.32
1891...	3.68	3.81	3.85	3.89	3.69	3.64	3.44	3.37	3.43	3.38	3.49	3.90	3.63
1892...	3.90	4.18	4.31	4.30	3.97	3.59	3.56	3.44	3.39	3.49	3.66	3.94	3.81
1893...	4.22	4.31	4.26	4.16	3.66	3.14	2.84	2.78	2.87	2.90	3.06	3.29	3.46
1894...	3.47	3.65	3.56	3.41	2.63	2.41	2.35	2.22	2.30	2.28	2.33	2.52	2.76
1895...	2.82	3.04	3.21	3.31	2.94	2.62	2.42	2.37	2.23	2.18	2.47	2.80	2.70
Mean 25 yrs. 1871-95	3.48	3.63	3.69	3.68	3.38	3.07	2.86	2.78	2.76	2.79	2.94	3.22	3.19

TABLE No. 1—(*Continued*).

Lake Michigan, at Milwaukee, Wis.

Monthly mean of water levels below the plane of reference of United States Lake Survey (high water of 1838).

Elevation of the plane of reference above mean tide at New York City is 584.34 ft.

Year.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Mean
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
1871...	3.12	3.20	2.60	2.40	2.05	2.01	1.98	2.21	2.88	3.57	3.62	4.21	2.82
1872...	4.34	4.34	4.56	4.31	4.06	3.69	3.66	3.68	3.75	3.87	4.16	4.82	4.10
1873...	4.82	4.78	4.47	3.90	3.34	2.71	2.75	2.65	2.84	2.90	3.13	3.17	3.45
1874...	3.21	2.92	2.77	2.87	2.89	2.52	2.59	2.58	2.83	3.18	3.38	3.72	2.96
1875...	3.92	3.99	3.93	3.57	3.01	2.77	2.80	2.63	2.70	2.85	3.06	3.25	3.21
1876...	3.30	3.10	2.77	2.57	1.95	1.54	1.20	1.27	1.32	1.90	1.80	2.27	2.08
1877...	2.41	2.40	2.40	2.02	2.13	2.06	2.09	2.21	2.42	2.41	2.53	2.59	2.31
1878...	2.71	2.78	2.62	2.60	2.30	2.16	2.15	2.47	2.67	2.78	2.91	3.23	2.62
1879...	3.54	3.53	3.49	3.50	3.37	3.30	3.21	3.40	3.52	3.74	3.96	3.93	3.54
1880...	3.89	3.98	3.94	3.77	3.43	2.89	2.67	2.64	2.94	3.28	3.60	3.77	3.40
1881...	3.76	3.55	3.26	2.35	2.84	2.61	2.64	2.64	2.87	2.54	2.71	2.81	2.88
1882...	3.03	3.04	2.67	2.54	2.44	2.17	2.11	1.92	2.04	2.45	2.66	2.99	2.51
1883...	3.25	3.21	3.12	2.91	2.43	2.07	1.47	1.50	1.69	1.91	2.36	2.44	2.36
1884...	2.66	2.54	2.29	2.11	1.90	1.74	1.90	2.04	2.29	2.29	2.65	2.68	2.26
1885...	2.67	2.44	2.48	2.29	1.93	1.72	1.63	1.42	1.56	1.70	2.00	2.29	2.01
1886...	2.06	2.04	1.76	1.49	1.23	1.16	1.35	1.58	1.82	1.92	2.26	2.59	1.77
1887...	2.67	2.30	2.14	2.19	1.99	1.86	1.92	2.06	2.40	2.85	3.18	3.30	2.41
1888...	3.48	3.53	3.35	3.14	2.76	2.49	2.48	2.60	2.75	3.00	3.05	3.63	3.02
1889...	3.65	3.68	3.70	3.69	3.61	3.15	2.97	3.21	3.38	3.63	3.98	4.16	3.57
1890...	4.08	4.12	4.14	3.82	3.59	3.18	3.11	3.19	3.39	3.50	3.84	4.19	3.68
1891...	4.21	4.45	4.26	3.95	3.85	3.70	3.87	3.94	4.17	4.53	4.93	4.99	4.24
1892...	4.87	4.68	4.78	4.72	4.30	3.85	3.84	3.76	3.96	4.20	4.47	4.74	4.35
1893...	4.75	4.61	4.50	4.04	3.74	3.41	3.39	3.56	3.88	4.02	4.41	4.48	4.07
1894...	4.47	4.44	4.18	4.08	3.49	3.33	3.30	3.38	3.81	4.02	4.29	4.64	3.95
1895...	4.82	4.93	4.96	4.76	4.60	4.55	4.66	4.78	5.05	5.42	5.64	5.75	4.99
Mean 25 yrs. 1871–95	3.59	3.54	3.41	3.18	2.93	2.67	2.63	2.68	2.92	3.14	3.39	3.63	3.14

TABLE No. 1—(*Continued*).

Lake Erie, at Cleveland, Ohio.

Monthly mean of water levels below the plane of reference of United States Lake Survey (high water of 1838).

Elevation of the plane of reference above mean tide at New York City is 575.20 ft.

Year.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Mean
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
1871...	2.66	2.99	2.54	2.06	1.79	1.76	1.78	1.99	2.16	2.83	3.01	3.45	2.42
1872...	3.53	3.77	3.86	3.66	3.22	2.85	2.86	2.89	3.12	3.29	3.62	3.85	3.38
1873...	3.95	3.94	3.87	2.59	1.92	1.84	1.86	1.92	2.32	2.62	2.82	2.45	2.67
1874...	2.06	2.01	1.98	1.81	1.72	1.65	1.62	1.78	2.24	2.68	3.10	3.31	2.16
1875...	3.54	3.71	3.57	3.17	2.70	2.27	2.14	2.15	2.29	2.78	2.93	2.71	2.83
1876...	2.75	2.19	1.54	1.02	0.70	0.59	0.70	1.00	1.17	1.70	1.62	1.96	1.41
1877...	2.36	2.52	2.75	2.32	2.07	1.99	1.75	1.89	1.97	2.37	2.45	2.37	2.23
1878...	2.29	2.15	2.02	1.60	1.34	1.34	1.34	1.58	1.71	2.06	2.26	2.18	1.82
1879...	2.60	2.74	2.71	2.35	2.20	2.11	2.08	2.30	2.63	2.86	3.33	3.07	2.58
1880...	2.57	2.53	2.39	2.23	1.96	1.85	1.76	2.00	2.23	2.67	2.75	3.09	2.34
1881...	3.50	3.39	3.07	2.37	1.97	1.73	1.78	2.10	2.45	2.50	2.68	2.47	2.50
1882...	2.00	2.00	1.55	1.33	1.13	0.98	1.05	1.19	1.46	1.91	2.23	2.74	1.63
1883...	2.83	2.62	2.43	2.31	1.85	1.15	0.95	1.01	1.32	1.64	2.02	1.99	1.84
1884...	2.32	2.06	1.87	1.32	1.05	0.97	1.19	1.35	1.78	2.11	2.59	2.66	1.77
1885...	2.84	3.05	3.19	2.37	1.64	1.13	1.17	1.16	1.31	1.41	1.53	1.58	1.87
1886...	1.56	2.29	2.48	1.60	1.30	1.20	1.22	1.43	1.67	1.90	2.31	2.26	1.77
1887...	2.49	2.07	1.26	1.24	1.06	1.03	1.27	1.59	1.82	2.41	2.68	2.66	1.80
1888...	2.84	3.11	3.01	2.38	2.13	2.00	1.85	1.95	2.39	2.76	2.70	2.82	2.50
1889...	2.80	2.96	3.12	2.77	2.59	2.16	1.96	2.27	2.66	3.08	3.35	3.09	2.73
1890...	2.73	2.44	2.32	1.83	1.49	1.12	1.50	1.94	2.13	2.32	2.35	2.58	2.06
1891...	2.80	2.82	2.36	2.49	2.67	2.53	2.63	2.90	3.08	3.46	3.90	3.83	2.96
1892...	3.80	4.01	3.97	3.41	2.61	1.85	1.73	2.08	2.40	2.96	3.29	3.56	2.97
1893...	3.94	3.86	3.64	2.91	2.07	1.88	2.16	2.50	2.88	3.23	3.63	3.55	3.02
1894...	3.27	3.39	3.36	2.96	2.57	2.26	2.38	2.75	2.92	3.24	3.48	3.55	3.01
1895...	3.88	4.11	4.10	3.85	3.63	3.54	3.65	3.73	3.83	4.31	4.41	4.25	3.94
Mean 25 yrs. 1871–95	2.88	2.91	2.76	2.32	1.96	1.75	1.78	1.98	2.24	2.60	2.84	2.88	2.41

TABLE No. 1—(*Continued*).

Lake Ontario, at Oswego, New York.

Monthly mean of water levels above the zero of the United States Engineer gauge, the elevation of which zero is 244.21 ft. above mean tide at New York City.

Year.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Mean
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
1871...	1.94	1.77	1.98	2.58	3.00	2.94	2.78	2.34	2.00	1.50	1.09	0.78	2.06
1872...	0.61	0.39	0.23	0.72	0.84	1.17	1.23	1.07	0.78	0.62	0.57	0.23	0.71
1873...	0.19	0.26	0.38	2.34	2.88	2.80	2.76	2.48	2.06	1.61	1.48	1.67	1.74
1874...	2.23	2.63	3.18	3.07	3.05	3.14	3.09	2.86	2.22	1.82	1.25	0.91	2.45
1875...	0.61	0.26	0.52	1.32	1.59	1.75	1.78	1.64	1.43	1.15	0.96	0.78	1.15
1876...	1.19	1.85	2.40	3.38	3.96	4.18	4.25	3.79	3.18	2.84	2.48	2.30	2.98
1877...	1.77	1.50	1.65	2.34	2.41	2.31	2.35	2.08	1.65	1.22	1.13	1.26	1.81
1878...	1.36	1.57	2.27	2.52	2.86	2.85	2.81	2.73	2.47	2.21	2.09	2.90	2.39
1879...	2.69	2.33	2.18	2.59	2.68	2.71	2.55	2.20	1.78	1.34	0.95	0.98	2.08
1880...	1.20	1.48	1.82	2.00	2.15	2.39	2.40	1.97	1.60	1.19	1.15	0.98	1.69
1881...	0.62	0.61	1.27	1.69	1.87	2.09	2.16	1.84	1.28	1.06	1.06	1.06	1.38
1882...	1.61	1.78	2.38	2.71	2.90	3.41	3.40	3.07	2.69	2.18	1.76	1.47	2.45
1883...	1.20	1.26	1.50	2.02	2.67	3.37	3.90	3.72	3.24	2.80	2.57	2.43	2.56
1884...	2.39	2.76	3.44	4.05	4.07	3.97	3.76	3.53	3.10	2.68	2.18	2.03	3.16
1885...	2.02	1.75	1.47	2.15	2.95	3.32	3.46	3.31	3.09	2.90	2.95	3.12	2.71
1886...	3.48	3.55	3.69	4.31	4.52	4.32	3.92	3.48	3.12	2.83	2.39	2.30	3.49
1887...	2.05	2.80	3.31	3.52	4.08	4.04	3.76	3.25	2.64	2.25	1.90	1.63	2.94
1888...	1.32	1.18	1.42	2.05	2.12	2.16	2.22	2.12	1.73	1.37	1.30	1.29	1.69
1889...	1.50	1.64	1.81	2.05	2.20	2.51	2.70	2.45	1.89	1.45	1.05	1.62	1.91
1890...	2.13	2.48	2.81	3.05	3.41	4.04	3.87	3.20	2.85	2.52	2.60	2.39	2.95
1891...	2.07	2.33	2.87	3.35	3.13	2.71	2.43	1.99	1.56	0.92	0.32	0.29	2.00
1892...	0.39	0.36	0.49	1.07	1.13	1.69	2.20	2.12	1.92	1.48	1.21	1.08	1.26
1893...	0.75	0.64	1.12	1.87	3.08	3.25	2.99	2.45	2.18	1.66	1.25	1.10	1.86
1894...	1.44	1.62	1.92	1.97	2.15	2.68	2.48	1.90	1.39	1.14	0.81	0.46	1.66
1895...	0.37	0.31	0.21	0.76	0.88	0.76	0.47	+0.22	—0.12	—0.46	—0.71	—0.68	0.17
Mean 25 yrs. 1871-95	1.49	1.56	1.85	2.38	2.66	2.82	2.79	2.47	2.07	1.69	1.43	1.38	2.05

TABLE No. 2.—Monthly Changes in Water Level of the Great Cubic Feet per Second per Month, and the Cum-

By cumulative effect is here meant the increase or decrease in the level in the

	Superior.		Michigan-Huron.			Erie.			Ontario.		
	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect.—Superior—Michigan-Huron.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect.—Superior—Michigan-Huron—Erie.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect.—Superior—Michigan-Huron—Erie—Ontario.
1871.											
January	−0.09	− 30	+0.15	+ 71	+ 41	−0.21	− 22	+ 19	−0.07	− 5	+ 14
February	−0.60	−209	−0.08	− 40	−249	−0.33	− 36	−285	−0.17	− 14	−299
March	+0.42	+146	+0.60	+299	+445	+0.45	+ 49	+494	+0.21	− 17	+511
April	+0.50	+168	+0.20	+ 96	+264	+0.48	+ 51	+315	+0.60	+ 47	+302
May	+0.53	+178	+0.35	+169	+347	+0.27	+ 29	+376	+0.42	+ 33	+409
June	+0.12	+ 40	+0.04	+ 19	+ 59	+0.03	+ 3	+ 62	−0.06	− 5	+ 57
July	+0.07	+ 24	+0.03	+ 14	+ 38	−0.02	− 2	+ 36	−0.16	− 13	+ 23
August	+0.06	+ 20	−0.23	−109	− 89	−0.21	− 22	−111	−0.44	− 34	−145
September	+0.10	+ 34	−0.67	−323	−289	−0.17	− 18	−307	−0.34	− 27	−334
October	−0.07	− 24	−0.69	−333	−357	−0.67	− 71	−428	0.50	− 39	−467
November	−0.07	− 24	−0.05	− 24	− 48	−0.18	− 19	− 67	−0.41	− 32	− 99
December	−0.74	−249	−0.59	−285	−534	−0.44	− 47	−581	−0.31	− 24	−605
1872.											
January	−0.32	−106	−0.13	− 62	−168	−0.08	− 8	−176	−0.17	− 13	−189
February	−0.11	− 38	0.00	00	− 38	−0.24	− 26	− 64	−0.22	− 17	− 81
March	−0.12	− 41	−0.22	−108	−149	−0.09	− 10	−159	−0.16	− 13	−172
April	−0.10	− 34	+0.25	+121	+ 87	+0.20	+ 21	+108	+0.49	+ 39	+147
May	+0.65	+219	+0.25	+121	+340	+0.44	+ 47	+387	+0.12	+ 9	+396
June	+0.38	+128	+0.37	+178	+306	+0.37	+ 39	+345	+0.33	+ 26	+371
July	+0.27	+ 91	+0.08	+ 14	+105	−0.01	− 1	+104	+0.06	+ 5	+109
August	+0.17	+ 56	−0.02	− 9	+ 47	−0.08	− 3	+ 44	−0.16	− 12	+ 32
September	+0.16	+ 54	−0.07	− 34	+ 20	−0.23	− 24	− 4	−0.29	− 23	− 27
October	−0.10	− 34	−0.12	− 58	− 92	−0.17	− 18	−110	−0.16	− 13	−123
November	−0.15	− 50	−0.29	−140	−190	−0.33	− 35	−225	−0.05	− 4	−229
December	−0.30	−101	−0.66	−318	−419	−0.23	− 24	−443	−0.34	− 27	−470
1873.											
January	− 0.10	− 33	0.00	000	− 33	−0.10	− 10	− 43	−0.04	− 3	− 46
February	−0.45	−156	+0.04	+ 20	−136	+0.01	+ 1	−135	+0.07	+ 6	−129
March	+0.03	+ 10	+0.31	+155	+165	+0.07	+ 8	+173	+0.12	+ 10	+183
April	+0.01	+ 3	+0.57	+275	+278	−1.28	+135	+413	+1.96	+154	+567
May	+0.60	+202	+0.56	+270	+472	+0.67	+ 71	+543	+0.54	+ 43	+586
June	+0.30	+101	+0.63	+304	+405	+0.08	+ 8	+413	−0.08	− 6	+407
July	+0.29	+ 98	−0.04	− 19	+ 79	−0.02	− 2	+ 77	−0.04	− 3	+ 74
August	+0.18	+ 60	+0.10	+ 47	+107	−0.06	− 6	+101	−0.28	− 22	+ 79
September	+0.06	+ 20	−0.19	− 92	− 72	−0.40	− 42	−114	−0.42	− 33	−147
October	−0.10	− 34	−0.06	− 29	− 63	−0.30	− 32	− 95	−0.45	− 35	−130
November	−0.14	− 47	−0.23	−111	−158	−0.20	− 21	−179	−0.13	− 10	−189
December	−0.30	−101	−0.04	− 19	−120	+0.37	+ 39	− 81	+0.19	+ 15	− 66
1874.											
January	−0.45	−149	−0.04	− 19	−168	+0.39	+ 41	−127	+0.56	+ 43	− 84
February	−0.02	− 7	+0.29	+145	+138	+0.05	+ 5	+143	+0.40	+ 33	+176
March	−0.04	− 14	+0.15	+ 75	+ 61	+0.03	+ 3	+ 64	+0.55	+ 45	+109
April	+0.10	+ 34	−0.10	− 48	− 14	+0.17	+ 18	+ 4	−0.11	− 9	− 5
May	+0.07	+ 24	−0.02	− 10	+ 14	+0.09	+ 10	+ 24	−0.02	− 2	+ 22
June	+0.20	+ 67	+0.37	+178	+245	+0.07	+ 7	+252	+0.09	+ 7	+259
July	+0.38	+128	−0.07	− 34	+ 94	+0.03	+ 3	+ 97	−0.05	− 4	+ 93
August	+0.09	+ 30	+0.01	+ 5	+ 35	−0.16	− 17	+ 18	−0.23	− 18	000
September	+0.10	+ 34	−0.25	−121	− 87	−0.46	− 49	−136	−0.64	− 50	−186
October	+0.06	+ 20	−0.35	−169	−149	−0.44	− 47	−196	−0.40	− 32	−228
November	−0.18	− 61	−0.20	− 96	−157	−0.42	− 44	−201	−0.57	− 45	−246
December	−0.31	−104	−0.34	−164	−268	−0.21	− 22	−290	−0.34	− 27	−317

LAKES, WITH CORRESPONDING RATES OF STORAGE IN THOUSANDS OF ULATIVE EFFECTS IN THE LAKES BELOW SUPERIOR.

normal discharge of any outlet by the elimination of oscillations of lakes above.

	SUPERIOR.		MICHIGAN-HURON.			ERIE.			ONTARIO.		
	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect,—Superior—Michigan—Huron.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect,—Superior—Michigan—Huron—Erie.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect,—Superior—Michigan—Huron—Erie—Ontario.
1875.											
January	−0.32	−106	−0.20	− 95	−201	−0.23	− 24	−225	−0.30	− 23	−248
February	−0.04	− 14	−0.07	− 35	− 49	−0.17	− 19	− 68	−0.35	− 29	− 97
March	+0.04	+ 14	+0.06	+ 30	+ 44	+0.14	+ 15	+ 59	+0.26	+ 21	+ 80
April	0.00	.....	+0.36	+174	+174	+0.40	+ 42	+216	+0.80	+ 63	+279
May	+0.22	+ 74	+0.56	+270	+344	+0.47	+ 50	+394	+0.27	+ 21	+415
June	+0.36	+121	+0.24	+116	+237	+0.43	+ 45	+282	+0.16	+ 13	+295
July	−0.01	− 3	−0.08	− 14	− 17	+0.13	+ 14	− 3	+0.03	+ 2	− 1
August	+0.09	+ 30	+0.17	+ 81	+111	−0.01	− 1	+110	−0.14	− 11	+ 99
September	+0.22	+ 77	−0.07	− 34	+ 43	−0.14	− 15	+ 28	−0.21	− 17	+ 11
October	−0.15	− 50	−0.15	− 72	−122	−0.49	− 52	−174	−0.28	− 22	−196
November	−0.14	− 47	−0.21	−101	−148	−0.15	− 16	−164	−0.19	− 15	−179
December	−0.20	− 67	−0.19	− 92	−159	+0.22	+ 23	−136	−0.18	− 14	−150
1876.											
January	−0.20	− 66	−0.05	− 24	− 90	−0.04	− 4	− 94	+0.41	+ 32	− 62
February	−0.21	− 72	+0.20	+ 98	+ 26	+0.56	+ 60	+ 86	+0.66	+ 53	+139
March	−0.09	− 30	+0.33	+102	+132	+0.65	+ 70	+202	+0.55	+ 44	+246
April	+0.03	+ 10	+0.20	+ 96	+106	+0.52	+ 55	+161	+0.98	+ 77	+238
May	−0.54	−182	+0.62	+299	+481	+0.32	+ 34	+515	+0.58	+ 46	+561
June	+0.68	+229	+0.41	+198	+427	+0.11	+ 12	+439	+0.22	+ 17	+456
July	+0.39	+131	+0.34	+164	+295	−0.11	− 12	+283	+0.07	+ 6	+289
August	+0.11	+ 36	−0.07	− 33	+ 3	−0.30	− 31	− 28	−0.46	− 36	− 64
September	−0.11	− 37	−0.05	− 24	− 61	−0.17	− 18	− 79	−0.61	− 48	−127
October	−0.33	−111	−0.58	−280	−391	−0.53	− 56	−447	−0.34	− 27	−474
November	−0.16	− 54	+0.10	+ 48	− 6	+0.08	+ 8	+ 2	−0.36	− 28	− 26
December	−0.28	− 94	−0.47	−227	−321	−0.34	− 36	−357	−0.18	− 14	−371
1877.											
January	−0.36	−119	−0.14	− 66	−185	−0.40	− 42	−227	−0.53	− 40	−267
February	−0.24	− 83	+0.01	+ 5	− 78	−0.16	− 17	− 95	−0.27	− 22	−117
March	−0.26	− 90	0.00	.....	− 90	−0.23	− 25	−115	+0.15	+ 12	−103
April	−0.08	− 27	+0.38	+183	+156	+0.43	+ 45	+201	+0.69	+ 54	+255
May	−0.01	− 3	−0.11	− 53	− 56	+0.25	+ 26	− 30	+0.07	+ 6	− 24
June	+0.22	+ 74	+0.07	+ 34	+108	+0.08	+ 8	+116	−0.10	− 8	+108
July	+0.38	+128	−0.03	− 14	+114	+0.24	+ 25	+139	+0.04	+ 3	+142
August	+0.06	+ 20	−0.12	− 57	− 37	−0.14	− 15	− 52	−0.27	− 21	− 73
September	−0.16	− 54	−0.21	−101	−155	−0.08	− 8	−163	−0.43	− 34	−197
October	0.00	.....	+0.01	+ 5	+ 5	−0.40	− 42	− 37	−0.43	− 34	− 71
November	−0.21	− 71	−0.12	− 58	−129	−0.08	− 8	−137	−0.09	− 7	−144
December	−0.07	− 24	−0.06	− 29	− 53	+0.08	+ 8	− 45	+0.13	+ 10	− 35
1878.											
January	−0.12	− 40	−0.12	− 57	− 97	+0.08	+ 8	− 89	+0.10	+ 8	− 81
February	+0.12	+ 42	−0.07	− 35	+ 7	+0.14	+ 15	+ 22	+0.21	+ 17	+ 39
March	−0.77	−268	+0.16	+ 80	−188	+0.13	+ 14	−174	+0.70	+ 57	−117
April	−0.03	− 10	+0.02	+ 10	00	+0.42	+ 44	+ 44	+0.25	+ 20	+ 64
May	+0.27	+ 91	+0.30	+145	+236	+0.26	+ 27	+263	+0.34	+ 27	+290
June	+0.28	+ 94	+0.14	+ 68	+162	0.00	.....	+162	−0.01	− 1	+161
July	+0.07	+ 24	+0.01	+ 5	+ 29	0.00	.....	+ 29	−0.04	− 3	+ 26
August	−0.12	− 40	−0.32	−152	−192	−0.24	− 25	−217	−0.08	− 6	−223
September	−0.17	− 57	−0.20	− 96	−153	−0.13	− 14	−167	−0.26	− 20	−187
October	+0.07	+ 24	−0.11	− 53	− 29	−0.35	− 37	− 66	−0.26	− 20	− 86
November	−0.20	− 67	−0.13	− 63	−130	−0.20	− 21	−151	−0.12	− 9	−160
December	−0.32	−104	−0.32	−159	−263	+0.08	+ 8	−255	+0.81	+ 64	−191

TABLE No. 2—

	Superior.		Michigan-Huron.			Erie.			Ontario.		
	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect.—Superior—Michigan-Huron.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect.—Superior—Michigan-Huron—Erie.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect.—Superior—Michigan-Huron—Erie—Ontario.
1879.											
January	+0.10	+ 33	−0.31	−147	−114	−0.42	− 44	−158	−0.21	− 16	−174
February	−0.04	− 14	+0.01	+ 5	− 9	−0.14	− 15	− 24	−0.36	− 29	− 53
March	+0.30	+104	+0.04	+ 20	+124	+0.03	+ 3	+127	−0.15	− 12	+115
April	−0.39	−131	−0.01	− 5	−136	+0.36	+ 38	− 98	+0.41	+ 32	− 66
May	−0.36	−121	+0.13	+ 63	− 58	+0.15	+ 16	− 42	+0.09	+ 7	− 35
June	+0.23	+ 77	+0.07	+ 34	+111	+0.09	+ 10	+121	+0.03	+ 2	+123
July	+0.24	+ 81	+0.09	+ 43	+124	+0.03	+ 3	+127	−0.16	− 13	+114
August	+0.12	+ 40	−0.19	− 90	− 50	−0.22	− 23	− 73	−0.35	− 27	−100
September	−0.11	− 37	−0.12	− 58	− 95	−0.33	− 35	−130	−0.42	− 33	−163
October	+0.09	+ 30	−0.22	−106	− 76	−0.23	− 24	−100	−0.44	− 35	−135
November	−0.08	− 27	−0.22	−106	−133	−0.47	− 50	−183	−0.39	− 31	−214
December	−0.30	−101	+0.03	+ 14	− 87	+0.26	+ 27	− 60	+0.03	+ 2	− 58
1880.											
January	−0.32	−106	+0.04	+ 19	− 87	+0.50	+ 52	− 35	+0.22	+ 17	− 18
February	−0.04	− 14	−0.09	− 44	− 58	+0.04	+ 4	− 54	+0.28	+ 22	− 32
March	−0.09	− 31	+0.04	+ 20	− 11	+0.14	+ 15	+ 4	+0.34	+ 27	+ 31
April	+0.03	+ 10	+0.17	+ 82	+ 92	+0.16	+ 17	+109	+0.18	+ 14	+123
May	+0.69	+232	+0.34	+164	+396	+0.27	+ 29	+425	+0.15	+ 12	+437
June	+0.83	+279	+0.54	+260	+539	+0.11	+ 12	+551	+0.24	+ 19	+570
July	+0.15	+ 50	+0.22	+106	+156	+0.09	+ 10	+166	+0.01	+ 1	+167
August	−0.01	− 3	+0.03	+ 14	+ 11	−0.24	− 25	− 14	−0.43	− 33	− 47
September	0.00	.....	−0.30	−145	−145	−0.23	− 24	−169	−0.37	− 29	−198
October	−0.05	− 17	−0.34	−164	−181	−0.44	− 47	−228	−0.41	− 32	−260
November	−0.06	− 20	−0.32	−154	−174	−0.08	− 8	−182	−0.14	− 11	−193
December	−0.26	− 87	−0.17	− 82	−169	−0.34	− 36	−205	−0.17	− 13	−218
1881.											
January	−0.26	− 86	+0.01	+ 5	− 81	−0.41	− 43	−124	−0.36	− 28	−152
February	−0.10	− 35	+0.21	+105	+ 70	+0.11	+ 12	+ 82	−0.01	− 1	+ 81
March	−0.09	− 31	+0.29	+145	+114	+0.32	+ 35	+149	+0.66	+ 54	+203
April	−0.09	− 30	−0.09	− 43	− 73	+0.70	+ 74	+ 1	+0.42	+ 33	+ 34
May	+0.30	+101	−0.49	−236	−135	+0.40	+ 42	− 93	+0.18	+ 14	− 79
June	+0.44	+148	+0.23	−111	+259	−0.24	− 25	+284	+0.22	+ 17	+301
July	+0.06	+ 20	−0.03	− 14	+ 6	−0.05	− 5	+ 1	+0.07	+ 6	+ 7
August	+0.05	+ 17	0.00	.....	+ 17	−0.32	− 33	− 16	−0.32	− 25	− 41
September	+0.23	+ 77	−0.23	−111	− 34	−0.35	− 37	− 71	−0.56	− 44	−115
October	+0.34	+114	+0.33	+159	+273	−0.05	− 5	+268	−0.22	− 17	+251
November	−0.07	− 24	−0.17	− 82	−106	−0.18	− 19	−125	0.00	.....	−125
December	−0.28	− 94	−0.10	− 48	−142	+0.21	+ 22	−120	0.00	.....	−120
1882.											
January	−0.35	−116	- 0.22	−104	−220	+0.47	+ 49	−171	+0.55	+ 43	−128
February	−0.25	− 87	−0.01	− 5	− 92	0.00	.....	− 92	+0.17	+ 14	− 78
March	−0.11	− 38	+0.37	+184	+146	+0.45	+ 49	+195	+0.60	+ 49	+244
April	−0.08	− 27	+0.13	+ 63	+ 36	+0.22	+ 23	+ 59	+0.33	+ 26	+ 85
May	+0.16	+ 54	+0.10	+ 48	+102	+0.20	+ 21	+123	+0.19	+ 15	+138
June	+0.02	+ 7	+0.27	+130	+137	+0.15	+ 16	+153	+0.51	+ 40	+193
July	+0.45	+151	+0.06	+ 29	+180	−0.07	− 7	+173	−0.01	− 1	+172
August	+0.12	+ 40	+0.19	+ 90	+130	−0.14	− 15	+115	−0.33	− 26	+ 89
September	+0.04	+ 13	−0.12	− 58	− 45	−0.27	− 29	− 74	−0.38	− 30	−104
October	−0.17	− 57	−0.41	−198	−255	−0.45	− 48	−303	−0.51	− 40	−343
November	−0.02	− 7	−0.21	−101	−108	−0.32	− 34	−142	−0.42	− 33	−175
December	−0.19	− 64	−0.33	−159	−223	−0.51	− 54	−277	−0.29	− 23	−300
1883.											
January	−0.23	− 76	−0.26	−123	−199	−0.09	− 9	−208	−0.27	− 21	−229
February	−0.29	−101	+0.04	+ 20	− 81	+0.21	+ 23	− 58	+0.06	+ 5	− 53
March	0.00	.....	+0.09	+ 45	+ 45	+0.19	+ 21	+ 66	+0.24	+ 20	+ 86
April	+0.25	+ 84	+0.21	+101	+185	+0.12	+ 13	+198	+0.52	+ 41	+239
May	+0.01	+ 3	+0.48	+231	+234	+0.46	+ 49	+283	+0.65	+ 51	+334
June	+0.10	+ 34	+0.36	+174	+208	+0.70	+ 74	+282	+0.70	+ 55	+337
July	+0.25	+ 84	+0.60	+289	+373	+0.20	+ 21	+394	+0.53	+ 42	+436
August	+0.02	+ 7	−0.03	− 14	− 7	−0.06	− 6	− 13	−0.18	− 14	− 27
September	−0.04	− 13	−0.19	− 92	−105	−0.31	− 33	−138	−0.48	− 38	−176
October	−0.20	− 67	−0.22	−106	−173	−0.32	− 34	−207	−0.44	− 35	−242
November	−0.15	− 50	−0.45	−217	−267	−0.38	− 40	−307	−0.23	− 18	−325
December	−0.11	− 37	−0.08	− 39	− 76	+0.03	+ 3	− 73	−0.14	− 11	− 84

(*Continued*).

	Superior.		Michigan-Huron.			Erie.			Ontario.		
	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect,—Superior—Michigan-Huron.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect,—Superior—Michigan-Huron—Erie.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect,—Superior—Michigan-Huron—Erie—Ontario.
1884.											
January	−0.07	− 23	−0.22	−104	−127	−0.33	− 34	−161	−0.04	− 3	−164
February	−0.03	− 10	+0.12	+ 59	+ 49	+0.26	+ 28	+ 77	+0.37	+ 30	+107
March	−0.12	− 41	+0.25	+123	+ 82	+0.19	+ 20	+102	+0.68	+ 54	+156
April	−0.29	− 98	+0.18	+ 87	− 11	+0.55	+ 58	+ 47	+0.61	+ 48	+ 95
May	+0.22	+ 74	+0.21	+101	+175	+0.27	+ 29	+204	+0.02	+ 2	+206
June	+0.20	+ 67	+0.16	+ 77	+144	+0.08	+ 8	+152	−0.10	− 8	+144
July	+0.14	+ 47	−0.16	− 77	− 30	−0.22	− 23	− 53	−0.21	− 17	− 70
August	+0.01	+ 3	−0.14	− 66	− 63	−0.16	− 17	− 80	−0.23	− 18	− 98
September	+0.27	+ 91	−0.25	−121	− 30	−0.43	− 45	− 75	−0.43	− 34	−109
October	+0.36	+121	0.00	.....	+121	−0.33	− 35	+ 86	−0.42	− 33	+ 53
November	−0.10	− 34	−0.36	−174	−208	−0.48	− 51	−259	−0.50	− 39	−298
December	−0.21	− 71	−0.03	− 14	− 85	−0.07	− 7	− 92	−0.15	− 12	−104
1885.											
January	−0.23	− 76	+0.01	+ 5	− 71	−0.18	− 19	− 90	−0.01	− 1	− 91
February	−0.18	− 63	+0.23	+115	+ 52	−0.21	− 23	+ 29	−0.27	− 22	+ 7
March	−0.08	− 28	−0.04	− 20	− 48	−0.14	− 15	− 63	−0.28	− 23	− 86
April	−0.05	− 17	+0.19	+ 92	+ 75	+0.82	+ 87	+162	+0.68	+ 54	+216
May	+0.33	+111	+0.36	+174	+285	+0.73	+ 77	+362	+0.80	+ 63	+425
June	+0.28	+ 94	+0.21	+101	+195	+0.51	+ 54	+249	+0.37	+ 29	+278
July	+0.24	+ 81	+0.09	+ 43	+124	−0.04	− 4	+120	+0.14	+ 11	+131
August	+0.12	+ 40	+0.21	+100	+140	+0.01	+ 1	+141	−0.15	− 12	+129
September	−0.07	− 24	−0.14	− 68	− 92	−0.15	− 16	−108	−0.22	− 17	−125
October	−0.17	− 57	−0.14	− 68	−125	−0.10	− 11	−136	−0.19	− 15	−151
November	−0.15	− 50	−0.30	−145	−195	−0.12	− 13	−208	+0.05	+ 4	−304
December	−0.33	−111	−0.29	−140	−251	−0.05	− 5	−256	+0.17	+ 13	−243
1886.											
January	−0.20	− 66	+0.23	+109	+ 43	+0.02	+ 2	+ 45	+0.36	+ 28	+ 73
February	−0.13	− 45	+0.02	+ 10	− 35	−0.73	− 80	−115	+0.07	+ 6	−109
March	−0.06	− 21	+0.28	+140	+119	−0.19	− 21	+ 98	+0.14	+ 11	+109
April	+0.09	+ 30	+0.27	+130	+160	+0.88	+ 93	+253	+0.62	+ 49	+302
May	+0.25	+ 84	+0.26	+125	+209	+0.30	+ 32	+241	+0.21	+ 17	+258
June	+0.14	+ 47	+0.07	+ 34	+ 81	+0.10	+ 11	+ 92	−0.20	− 16	+ 76
July	+0.07	+ 24	−0.19	− 92	− 68	−0.02	− 2	− 70	−0.40	− 32	−102
August	−0.09	− 30	−0.23	−109	−139	−0.21	− 22	−161	−0.44	− 34	−195
September	−0.02	− 7	−0.24	−116	−123	−0.24	− 25	−148	−0.36	− 28	−176
October	+0.10	+ 34	−0.10	− 48	− 14	−0.23	− 24	− 38	−0.29	− 23	− 61
November	−0.15	− 50	−0.34	−164	−214	−0.41	− 43	−257	−0.44	− 35	−292
December	−0.14	− 47	−0.33	−159	−206	+0.05	+ 5	−201	−0.09	− 7	−208
1887.											
January	−0.31	−103	−0.08	− 38	−141	−0.23	− 24	−165	−0.25	− 19	−184
February	+0.02	+ 7	+0.37	+184	+191	+0.42	+ 46	+237	+0.75	+ 61	+298
March	+0.31	+108	+0.16	+ 80	+188	+0.81	+ 89	+277	+0.51	+ 42	+319
April	+0.17	+ 57	−0.05	− 24	+ 33	+0.02	+ 2	+ 35	+0.21	+ 17	+ 52
May	−0.21	− 71	+0.20	+ 96	+ 25	+0.18	+ 19	+ 44	+0.56	+ 44	+ 88
June	+0.16	+ 54	+0.13	+ 63	+117	+0.03	+ 3	+120	−0.04	− 3	+117
July	+0.28	+ 94	−0.06	− 29	+ 65	−0.24	− 25	+ 40	−0.28	− 22	+ 18
August	+0.08	+ 26	−0.14	− 66	− 40	−0.32	− 33	− 73	−0.51	− 40	−113
September	−0.14	− 47	−0.34	−164	−211	−0.23	− 24	−235	−0.61	− 48	−283
October	−0.07	− 24	−0.45	−217	−241	−0.59	− 62	−303	−0.39	− 31	−334
November	−0.24	− 81	−0.33	−159	−240	−0.27	− 29	−269	−0.35	− 27	−296
December	−0.22	− 74	−0.12	− 58	−132	+0.02	+ 2	−130	−0.27	− 21	−151
1888.											
January	−0.11	− 36	−0.18	− 85	−121	−0.18	− 19	−140	−0.31	− 24	−164
February	+0.01	+ 3	−0.05	− 25	− 22	−0.27	− 29	− 51	−0.14	− 11	− 62
March	−0.07	− 24	+0.18	+ 88	+ 64	+0.10	+ 11	+ 75	+0.24	+ 19	+ 94
April	0.00	.....	+0.21	+101	+101	+0.63	+ 67	+168	+0.63	+ 50	+218
May	+0.47	+158	+0.38	+183	+341	+0.25	+ 26	+367	+0.07	+ 6	+373
June	+0.78	+262	+0.27	+130	+392	+0.13	+ 14	+406	+0.04	+ 3	+409
July	+0.19	+ 64	+0.01	+ 5	+ 69	+0.15	+ 16	+ 85	+0.06	+ 5	+ 90
August	+0.14	+ 46	−0.12	− 57	− 11	−0.10	− 10	− 21	−0.10	− 8	− 29
September	−0.05	− 17	−0.15	− 72	− 89	−0.44	− 47	−136	−0.39	− 31	−167
October	−0.09	− 30	−0.25	−121	−151	−0.37	− 39	−190	−0.36	− 28	−218
November	−0.14	− 47	−0.05	− 24	− 71	+0.06	+ 6	− 65	−0.07	− 6	− 71
December	−0.35	−118	−0.58	−280	−398	−0.12	− 13	−412	−0.01	− 1	−413

TABLE No. 2—

	Superior.		Michigan-Huron.			Erie.			Ontario.		
	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect.—Superior—Michigan-Huron.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect.—Superior—Michigan-Huron—Erie.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect.—Superior—Michigan-Huron—Erie Ontario
1889.											
January	−0.32	−106	−0.02	− 9	−115	+0.02	+ 2	−113	+0.21	+ 16	− 97
February	−0.22	− 77	−0.03	− 15	− 92	−0.16	− 17	−109	+0.14	+ 11	− 98
March	−0.17	− 59	−0.02	− 10	− 69	−0.16	− 17	− 86	+0.17	+ 14	− 72
April	+0.01	+ 3	+0.01	+ 5	+ 8	+0.35	+ 37	+ 45	+0.24	+ 19	+ 64
May	+0.35	+118	+0.08	+ 39	+157	+0.18	+ 19	+176	+0.15	+ 12	+188
June	+0.12	+ 40	+0.46	+222	+262	+0.43	+ 45	+307	+0.31	+ 24	+331
July	+0.19	+ 64	+0.18	+ 87	+151	+0.20	+ 21	+172	+0.19	+ 15	+187
August	+0.19	+ 63	−0.24	−114	− 51	−0.31	− 32	− 83	−0.25	− 19	−102
September	+0.13	+ 44	−0.17	− 82	− 38	−0.39	− 41	− 79	−0.56	− 44	−123
October	−0.16	− 54	−0.25	−121	−175	−0.42	− 44	−219	−0.44	− 35	−254
November	−0.31	−104	−0.35	−169	−273	−0.27	− 29	−302	−0.40	− 32	−334
December	−0.30	−101	−0.18	− 87	−188	+0.26	+ 27	−161	+0.57	+ 45	−116
1890.											
January	−0.14	− 46	+0.08	+ 38	− 8	+0.36	+ 37	+ 29	+0.51	+ 40	+ 69
February	−0.13	− 45	−0.04	− 20	− 65	+0.29	+ 32	− 33	+0.35	+ 29	− 4
March	−0.24	− 83	−0.02	− 10	− 93	+0.12	+ 13	− 80	+0.33	+ 27	− 53
April	−0.03	− 10	+0.32	+154	+144	+0.49	+ 52	+196	+0.24	+ 19	+215
May	+0.21	+ 71	+0.23	+111	+182	+0.34	+ 36	+218	+0.36	+ 28	+246
June	+0.45	+151	+0.41	+198	+349	+0.37	+ 39	+388	+0.63	+ 50	+438
July	+0.30	+101	+0.07	+ 34	+135	−0.38	− 40	+ 95	−0.17	− 13	+ 82
August	+0.15	+ 50	−0.08	− 38	+ 12	−0.44	− 46	− 34	−0.67	− 52	− 86
September	+0.13	+ 44	−0.20	− 96	− 52	−0.19	− 20	− 72	−0.35	− 28	−100
October	−0.03	− 10	−0.11	− 53	− 63	−0.19	− 20	− 83	−0.33	− 26	−109
November	−0.21	− 71	−0.34	−164	−235	−0.03	− 3	−238	+0.08	+ 6	−232
December	−0.36	−121	−0.35	−169	−290	−0.23	− 24	−314	−0.21	− 17	−331
1891.											
January	−0.36	−119	−0.02	− 9	−128	−0.22	− 23	−151	−0.32	− 25	−176
February	−0.13	− 45	−0.24	−120	−165	−0.02	− 2	−167	+0.26	+ 21	−146
March	−0.04	− 14	+0.19	+ 95	+ 81	+0.46	+ 50	+131	+0.54	+ 44	+175
April	−0.04	− 13	+0.31	+150	+137	−0.13	− 14	+123	+0.48	+ 38	+161
May	+0.20	+ 67	+0.10	+ 48	+115	−0.18	− 19	+ 96	−0.22	− 17	+ 79
June	+0.05	+ 17	+0.15	+ 72	+ 89	+0.14	+ 15	+104	−0.42	− 33	+ 71
July	+0.20	+ 67	−0.17	− 82	− 15	−0.10	− 11	− 26	−0.28	− 22	− 48
August	+0.07	+ 23	−0.07	− 33	− 10	−0.27	− 28	− 38	−0.44	− 34	− 72
September	−0.06	− 20	−0.23	−111	−131	−0.18	− 19	−150	−0.43	− 34	−184
October	+0.05	+ 17	−0.36	−174	−157	−0.38	− 40	−197	−0.64	− 50	−247
November	−0.11	− 37	−0.40	−193	−230	−0.44	− 47	−277	−0.60	− 47	−324
December	−0.41	−139	−0.06	− 29	−168	+0.07	+ 7	−161	−0.08	− 2	−163
1892.											
January	0.00	.....	+0.12	+ 57	+ 57	+0.03	+ 3	+ 60	+0.10	+ 8	+ 68
February	−0.28	− 96	+0.19	+ 93	− 3	−0.21	− 23	− 26	−0.03	− 2	− 28
March	−0.13	− 44	−0.10	− 49	− 93	+0.04	+ 4	− 89	+0.13	+ 10	− 79
April	+0.01	+ 3	+0.06	+ 29	+ 32	+0.56	+ 58	+ 90	+0.58	+ 46	+136
May	+0.33	+111	+0.42	+203	+314	+0.80	+ 85	+399	+0.06	+ 5	+404
June	+0.38	+128	+0.45	+217	+345	+0.76	+ 80	+425	+0.56	+ 44	+469
July	+0.03	+ 10	+0.01	+ 5	+ 15	+0.12	+ 13	+ 28	+0.51	+ 40	+ 68
August	+0.12	+ 40	+0.08	+ 38	+ 78	−0.35	− 36	+ 42	−0.08	− 6	+ 36
September	+0.05	+ 17	−0.20	− 96	− 79	−0.32	− 34	−113	−0.20	− 16	−129
October	−0.10	− 34	−0.24	−116	−150	−0.56	− 59	−209	−0.44	− 35	−244
November	−0.17	− 57	−0.27	−130	−187	−0.38	− 35	−222	−0.27	− 21	−243
December	−0.28	− 94	−0.27	−130	−224	−0.27	− 29	−253	−0.13	− 10	−263

(*Continued*).

	Superior.		Michigan-Huron.			Erie.			Ontario.		
	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect,—Superior—Michigan-Huron.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect,—Superior—Michigan-Huron—Erie.	Fluctuation in feet.	Storage in 1 000 second-feet per month.	Cumulative effect,—Superior—Michigan-Huron—Erie—Ontario.
1893.											
January	−0.28	− 98	−0.01	− 5	− 98	−0.38	− 40	−138	−0.33	− 26	−164
February	−0.09	− 31	+0.14	+ 70	+ 39	+0.08	+ 9	+ 48	−0.11	− 9	+ 39
March	+0.05	+ 17	+0.11	+ 55	+ 72	+0.22	+ 24	+ 96	+0.48	+ 39	+135
April	+0.10	+ 34	+0.46	+222	+256	+0.73	+ 77	+333	+0.75	+ 59	+392
May	+0.50	+168	+0.30	+145	+313	+0.84	+ 89	+402	+1.16	+ 91	+493
June	+0.52	+175	+0.33	+159	+334	+0.19	+ 20	+354	+0.22	+ 17	+371
July	+0.30	+101	+0.02	+ 10	+111	−0.28	− 30	+ 81	−0.26	− 20	+ 61
August	+0.06	+ 20	−0.17	− 81	− 61	−0.34	− 35	− 96	−0.54	− 42	−138
September	−0.09	− 30	−0.32	−154	−184	−0.38	− 40	−224	−0.27	− 21	−245
October	−0.03	− 10	−0.14	− 68	− 78	−0.35	− 37	−115	−0.52	− 41	−156
November	−0.16	− 54	−0.39	−188	−242	−0.40	− 42	−284	−0.41	− 32	−316
December	−0.23	− 77	−0.07	− 34	−111	+0.08	+ 8	−103	−0.15	− 12	−115
1894.											
January	−0.18	− 60	+0.01	+ 5	− 55	+0.28	+ 29	− 26	+0.34	+ 26	0
February	−0.18	− 63	+0.03	+ 15	− 48	−0.12	− 13	− 61	+0.18	+ 15	− 46
March	+0.09	+ 31	+0.26	+130	+161	+0.03	+ 3	+164	+0.30	+ 24	+188
April	+0.15	+ 50	+0.15	+ 72	+122	+0.40	+ 42	+164	+0.05	+ 4	+168
May	+0.78	+262	+0.54	+260	+522	+0.39	+ 41	+563	+0.18	+ 14	+577
June	+0.22	+ 74	+0.16	+ 77	+151	+0.31	+ 33	+184	+0.53	+ 42	+226
July	+0.06	+ 20	+0.03	+ 14	+ 34	−0.12	− 13	+ 21	−0.20	− 16	+ 5
August	+0.13	+ 43	−0.08	− 38	+ 5	−0.37	− 39	− 34	−0.58	− 45	− 79
September	−0.08	− 27	−0.43	−207	−234	−0.17	− 18	−252	−0.51	− 40	−292
October	+0.02	+ 7	−0.21	−101	− 94	−0.32	− 34	−128	−0.25	− 20	−148
November	−0.05	− 17	−0.27	−130	−147	−0.24	− 25	−172	−0.33	− 26	−198
December	−0.19	− 64	−0.35	−169	−233	−0.07	− 7	−240	−0.35	− 28	−268
1895.											
January	−0.30	− 99	−0.18	− 85	−184	−0.33	− 34	−218	−0.09	− 7	−225
February	−0.22	− 77	−0.11	− 55	−132	−0.23	− 25	−157	−0.06	− 5	−162
March	−0.17	− 59	−0.03	− 15	− 74	+0.01	+ 1	− 73	−0.10	− 8	− 81
April	−0.10	− 34	+0.20	+ 96	+ 62	+0.25	+ 26	+ 88	+0.55	+ 43	+131
May	+0.37	+124	+0.16	+ 77	+201	+0.22	+ 23	+224	+0.12	+ 9	+233
June	+0.32	+108	+0.05	+ 24	+132	+0.09	+ 10	+142	−0.12	− 9	+133
July	+0.20	+ 67	−0.11	− 53	+ 14	−0.11	− 12	+ 2	−0.29	− 23	− 21
August	+0.05	+ 17	−0.12	− 57	− 40	−0.08	− 8	− 48	−0.25	− 19	− 67
September	+0.14	+ 47	−0.27	−130	− 83	−0.10	− 11	− 94	−0.34	− 27	−121
October	+0.05	+ 17	−0.37	−178	−161	−0.48	− 51	−212	−0.34	− 27	−239
November	−0.29	− 98	−0.22	−106	−204	−0.10	− 11	−215	−0.25	− 20	−235
December	−0.33	−111	−0.11	− 53	−164	+0.16	+ 17	−147	+0.03	+ 2	−145
Mean for 25 years. 1871-95.											
January	−0.26	− 86	−0.04	− 19	−105	0.00	0	−105	+0.11	+ 8	− 97
February	−0.15	− 52	+0.05	+ 25	− 27	−0.09	− 3	− 30	+0.07	+ 6	− 24
March	−0.06	− 21	+0.13	+ 65	+ 44	+0.15	+ 16	+ 60	+0.28	+ 22	+ 82
April	+0.01	+ 3	+0.23	+110	+113	+0.44	+ 46	+159	+0.53	+ 41	+200
May	+0.30	+101	+0.25	+120	+221	+0.36	+ 38	+259	+0.28	+ 21	+280
June	+0.31	+104	+0.26	+125	+229	+0.21	+ 22	+251	+0.16	+ 12	+263
July	+0.21	+ 70	+0.04	+ 19	+ 89	−0.03	− 3	+ 86	−0.03	− 2	− 84
August	+0.08	+ 26	−0.05	− 24	+ 2	−0.20	− 21	− 19	−0.32	− 25	− 44
September	+0.02	+ 7	−0.24	−115	−108	−0.26	− 27	−135	−0.40	− 32	−167
October	−0.03	− 10	−0.22	−106	−116	−0.36	− 38	−154	−0.38	− 30	−184
November	−0.15	− 50	−0.25	−120	−170	−0.24	− 25	−195	−0.26	− 21	−216
December	−0.28	− 94	−0.24	−115	−209	−0.04	− 4	−213	−0.05	− 4	−217

TABLE No. 3.—Oscillations in Mean Annual Levels of the Great per Second for each Period of One Year. The Table Effect at each Lake by Adding

Period.	Lake Superior.		Michigan-Huron.		
	Fluctuation.	Storage.	Fluctuation.	Storage for single lake.	Cumulative storage.
1870-71	−0.36	−10.1	−0.07	− 2.8	−12.9
1871-72	−0.04	− 1.1	−1.28	−51.6	−52.7
1872-73	+0.45	+12.6	+0.65	+26.2	+38.8
1873-74	+0.8	+ 2.2	+0.49	+19.8	+22.3
1874-75	+0.11	+ 3.1	−0.25	−10.1	− 7.0
1875-76	+0.39	+11.0	+1.13	+45.5	+56.5
1876-77	−0.62	−17.4	−0.23	− 9.3	−26.7
1877-78	−0.57	−16	−0.31	−12.5	−28.5
1878-79	−0.44	−13.8	−0.92	−37.1	−50.9
1879-80	+0.33	+ 9.3	+0.14	+ 5.6	+14.9
1880-81	+0.45	+12.6	+0.52	+21.0	+33.6
1881-82	0.00	0.0	+0.37	+14.9	+14.9
1882-83	−0.20	− 5.6	+0.15	+ 6.1	+ 0.3
1883-84	−0.11	− 3.1	+0.10	+ 4.0	+ 0.9
1884-85	+0.25	+ 7.0	+0.25	+10.1	+17.1
1885-86	−0.30	− 8.4	+0.24	+ 9.7	+ 1.3
1886-87	+0.03	+ 0.8	−0.64	−25.8	−25.0
1887-88	+0.40	+11.2	−0.61	−24.6	−13.4
1888-89	−0.14	− 3.9	−0.55	−22.2	−26.1
1889-90	−0.14	− 3.9	−0.11	− 4.4	− 8.3
1890-91	−0.31	− 8.7	−0.56	−22.6	−31.3
1891-92	−0.18	− 5.1	−0.11	− 4.4	− 9.5
1892-93	+0.35	+ 9.8	+0.28	+11.3	+21.1
1893-94	+0.70	+19.7	−0.12	− 4.8	+14.9
1894-95	+0.06	+ 1.7	−1.04	−41.9	−40.2

LAKES, AND THE EQUIVALENT STORAGE IN THOUSANDS OF CUBIC FEET GIVES THE VALUES FOR EACH LAKE AND THE CUMULATIVE THE STORAGE IN THOSE ABOVE.

Period.	ERIE.			ONTARIO.		
	Fluctuation.	Storage for single lake.	Cumulative storage.	Fluctuation.	Storage for single lake.	Cumulative storage.
1870-71.....	−0.59	− 5.2	−18.1	−1.41	− 9.3	−27.4
1871-72.....	−0.96	− 8.5	−61.2	−1.35	− 8.9	−70.1
1872-73.....	+0.71	+ 6.3	+45.1	+1.03	+ 6.8	+51.9
1873-74.....	+0.51	+ 4.5	+26.5	+0.71	+ 4.7	+31.2
1874-75.....	−0.67	− 5.9	−12.9	−1.30	− 8.6	−21.5
1875-76.....	+1.42	+12.6	+69.	+1.83	+12.1	+81.2
1976-77.....	−0.82	− 7.2	−33.9	−1.17	− 7.7	−41.6
1877-78.....	+0.41	+ 3.6	−24.9	+0.58	+ 3.8	−21.1
1878-79.....	−0.76	− 6.7	−57.6	−0.31	− 2.0	−59.6
1879-80.....	+0.24	+ 2.1	+17.0	−0.39	− 2.6	−14.4
1880-81.....	−0.16	− 1.4	+32.2	−0.31	− 2.0	+30.2
1881-82.....	+0.87	+ 7.7	+22.6	+1.07	+ 7.0	+33.3
1882-83.....	−0.21	− 1.9	− 1.6	+0.11	+ 0.7	− 0.5
1883-84.....	+0.07	+ 0.6	+ 1.5	+0.60	+ 4.0	+ 5.5
1884-85.....	−0.10	− 0.9	+16.2	−0.45	− 3.0	+13.2
1885-86.....	+0.10	+ 0.9	+ 2.2	+0.78	+ 5.1	+ 7.3
1886-87.....	−0.03	− 0.3	−25.3	−0.55	− 3.6	−28.9
1887-88.....	−0.70	− 6.2	−19.6	−1.25	− 8.2	−27.8
1888-89.....	−0.23	− 2.0	−28.1	+0.22	+ 1.5	−26.6
1889-90.....	+0.67	+ 5.9	− 2.4	+1.04	+ 6.9	+ 4.5
1890-91.....	−0.90	− 8.0	−39.3	−0.95	− 6.3	−45.6
1891-92.....	−0.01	− 0.1	− 9.6	−0.74	− 4.9	−14.5
1892-93.....	−0.05	− 0.4	+20.7	+0.60	+ 4.0	+24.7
1893-94.....	+0.01	+ 0.1	+15.0	−0.20	− 1.3	+13.7
1894-95.....	−0.93	− 8.2	−48.4	−1.49	− 9.8	−58.2

TABLE No. 4.—Storage in Thousands of Cubic Feet per Second Represented by a Uniform Rise or Fall of 1 Ft. and of 6 Ins. for Periods Varying from 1 to 6 Months, in the Great Lakes of the St. Lawrence Basin.

Time.	Lake Superior.		Lake Michigan-Huron.				Lake Erie.				Lake Ontario.			
			1 ft.		6 ins.		1 ft.		6 ins.		1 ft.		6 ins.	
	1 ft.	6 ins.	Single lake.	Cumulative effect.	Single lake.	Cumulative effect.	Single lake.	Cumulative effect.	Single lake.	Cumulative effect.	Single lake.	Cumulative effect.	Single lake.	Cumulative effect.
1 month....	336	168	482	818	241	409	106	924	53	462	79	1 003	39	501
2 months...	168	84	241	409	121	204	53	462	26	231	39	501	20	250
3 months...	112	56	161	273	80	136	35	308	18	154	26	334	13	167
4 months...	84	42	121	205	60	102	26	231	13	115	20	251	10	125
5 months...	67	34	96	164	48	82	21	185	11	92	16	201	8	100
6 months...	56	28	80	136	40	68	18	154	9	77	13	167	7	84

A MATHEMATICAL ANALYSIS OF THE INFLUENCE OF RESERVOIRS UPON STREAM-FLOW.

By JAMES A. SEDDON,* M. Am. Soc. C. E.

The fluctuation in the levels of a system of lakes only differs in its magnitude from the filling and emptying of a lock or a reservoir under varying conditions of inflow and outflow, though, of course, the true level over such large areas is not so easily observed. The action of the winds will certainly greatly change the levels at different points, the surface will answer to difference in barometric pressure, and small tidal effects may be found, and even after the primary disturbance is removed, the whole body of water may have been set to rocking backward and forward like a pendulum. It is thus only in the mean of a large number of points that the true level may be determined, or, as in actual practice, in the average of a time long enough to eliminate these shorter variations; but, this true level determined, all its variations are results alone of the inflow and the outflow.

The inflow will be called supply, and designated by S, and the outflow, discharge, and designated by Q, each taken in cubic feet per second, and with A representing the area of the lake in square feet, h its level on a scale of feet and T time in seconds. The fundamental equation of its fluctuation is

$$(S - Q)\, d\, T = A\, d\, h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (I)$$

In this equation, S, the supply of the lake, is made up of the run-off from the drainage basin surrounding it, which has its annual rise and fall, like the high and low-water periods of a river in a similar basin. But, unlike the river, the direct rainfall on its large area gives it at times a much more concentrated supply, and undoubtedly now and then its actual levels are subject to finite changes in comparatively insignificant intervals of time; but averaged up, as these are in the mean monthly levels, the effect of this direct supply approaches the annual cycle, and for a number of years must closely correspond to the variation of normal rainfall, and, finally, from these positive elements of supply there must be taken the negative element of evaporation, which has perhaps of all of them the most regular variation with the seasons.

* U. S. Assistant Engineer.

Thus, while the actual supply in a given year can never be exactly represented by any regular variation, the general type of supply may very fairly be assumed as a recurrent annual oscillation, and taken in its simplest form, as $S_1 + a \sin t$; where S_1 is a mean value of supply and t is a time arc on a circle, whose whole circumference represents one year, while as $\sin t$ passes from its extreme values of $+1$ to -1, the extreme oscillation of supply, of course, corresponds to the value of $2a$.

Unlike supply in its arbitrary character, Q, the discharge from the lake is, of course, determined at any time solely by the value of h, or the level of its outlet at that time. Just what relation exists between Q and h is a matter of observation. Were the outlet a plain weir of length l, with H measuring the height above its crest, the relation would be $Q = 3.3\, l\, H^{\frac{3}{2}}$, while the form that has more generally been applied to the flow of rivers is $Q = C\, H^2$; where H again gives the level above an approximate zero of discharge, and C is a constant to be determined from the discharge observations. Either of these more general expressions, however, makes the solution of Equation I unnecessarily difficult, and in the small oscillation of a lake surface it is sufficient to take Q as made up of a constant element Q_1, the value of discharge at an assumed level, and a variable element $c\,h$, where h gives the surface at any time above or below this arbitrary level, and c is the average change in discharge there per foot of rise or fall. Where large discharges are dealt with, some 10 or 15 ft. above their zero level, there is little difference in a foot or so between the arc of the discharge curve and the straight line $c\,h$, and it would be hardly necessary to consider this, even in the flow of a river; while in the case of the flow out of the lake, where the accidental oscillation at its outlet may exceed the whole range of its true level, such a difference would seem to be an even more needless refinement. In what follows, therefore, the discharge at any time will be taken as $Q_1 + c\,h$, or a linear variation with the value of h at that time.

Substituting these values for S and Q in Equation I gives

$$[(S_1 - Q_1) + (a \sin t - c\,h)]\, d\,T = A\, d\,h.$$

And for the present neglecting the arbitrary constant $(S_1 - Q_1)$, which may always be made zero at any time by taking a suitable level, from which h is to be measured for that time, Equation I becomes

$$(a \sin t - c\,h)\, d\,T = A\, d\,h.$$

In this equation T is time in seconds, while t is an arc on a circle in which 2π is to represent one year. The relation between them is, therefore, given by the following proportion:

$$T : t = 86\,400 \times 365 \text{ (seconds in one year)} : 2\pi$$

or $$T = \frac{86\,400 \times 365}{6.2832}\, t;$$

and $$d\,T = \frac{86\,400 \times 365}{6.2832}\, dt.$$

At the same time, it is also convenient to change the unit of A from square feet to square miles, so that the equation becomes

$$(a \sin t - c\,h)\,\frac{86\,400 \times 365}{6.2832}\,dt = (5\,280)^2\,A\,dh$$

or

$$(a \sin t - c\,h)\,0.180036\,dt = A\,dh \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (II)$$

or

$$dh + \frac{0.18\,c}{A}\,h\,dt = \frac{0.18}{A}\,a \sin t\,dt$$

This is a well-known form, and is solved by multiplying through by

$$e^{\frac{0.18\,c}{A}t}$$

or

$$e^{\frac{0.18\,c}{A}t}\,dh + h\,\frac{0.18\,c}{A}\,e^{\frac{0.18\,c}{A}t}\,dt = \frac{0.18}{A}\,a \sin t\, e^{\frac{0.18\,c}{A}t}\,dt$$

Where the left-hand side of the equation is the complete differential of

$$h \times e^{\frac{0.18\,c}{A}t},$$

and the right contains only the single variable t and is easily integrated by parts as follows:

$$\frac{0.18}{A}\,a \int \sin t\, e^{\frac{0.18\,c}{A}t}\,dt = \frac{0.18}{A}\,a \int \sin t\, \frac{d\left(e^{\frac{0.18\,c}{A}t}\right)}{\frac{0.18\,c}{A}}$$

$$= \frac{0.18}{A}\,a \left[\frac{\sin t\, e^{\frac{0.18\,c}{A}t}}{\frac{0.18\,c}{A}} - \frac{1}{\frac{0.18\,c}{A}} \int \cos t\, e^{\frac{0.18\,c}{A}t}\,dt\right]$$

And again:

$$\int \cos t\, e^{\frac{0.18\,c}{A}t}\,dt = \int \cos t\, \frac{d\left(e^{\frac{0.18\,c}{A}t}\right)}{\frac{0.18\,c}{A}}$$

$$= \frac{\cos t\, e^{\frac{0.18\,c}{A}t}}{\frac{0.18\,c}{A}} + \frac{1}{\frac{0.18\,c}{A}} \int \sin t\, e^{\frac{0.18\,c}{A}t}\, dt.$$

Hence

$$\frac{0.18}{A}\, a \int \sin t\, e^{\frac{0.18\,c}{A}t}\, dt = \frac{0.18}{A}\, a \left[\frac{\sin t\, e^{\frac{0.18\,c}{A}t}}{\frac{0.18\,c}{A}} - \frac{\cos t\, e^{\frac{0.18\,c}{A}t}}{\left(\frac{0.18\,c}{A}\right)^2} - \frac{1}{\left(\frac{0.18\,c}{A}\right)^2} \int \sin t\, e^{\frac{0.18\,c}{A}t}\, dt \right]$$

And solving for $\frac{0.18}{A}\, a \int \sin t\, e^{\frac{0.18\,c}{A}t}\, dt$, gives

$$\frac{0.18}{A}\, a \int \sin t\, e^{\frac{0.18\,c}{A}t}\, dt \left(1 + \frac{1}{\left(\frac{0.18\,c}{A}\right)^2}\right)$$

$$= \frac{0.18}{A}\, a\, \frac{1}{\left(\frac{0.18\,c}{A}\right)^2} \left[\frac{0.18\,c}{A} \sin t - \cos t\right] e^{\frac{0.18\,c}{A}t}$$

or, finally,

$$\frac{0.18}{A}\, a \int \sin t\, e^{\frac{0.18\,c}{A}t}\, dt = \frac{0.18}{A}\, a \left[\frac{\frac{0.18\,c}{A} \sin t - \cos t}{1 + \left(\frac{0.18\,c}{A}\right)^2} \right] e^{\frac{0.18\,c}{A}t}$$

And the primitive of differential Equation II becomes

$$h\, e^{\frac{0.18\,c}{A}t} = \frac{0.18}{A}\, a \left[\frac{\frac{0.18\,c}{A} \sin t - \cos t}{1 + \left(\frac{0.18\,c}{A}\right)^2} \right] e^{\frac{0.18\,c}{A}t} \pm C \left\{ \begin{array}{l} \text{The constant of integration.} \end{array} \right.$$

or dividing by $e^{\frac{0.18\,c}{A}t}$

$$h = \frac{0.18}{A}\, a \left[\frac{\frac{0.18\,c}{A} \sin t - \cos t}{1 + \left(\frac{0.18\,c}{A}\right)^2} \right] \pm \frac{C}{e^{\frac{0.18\,c}{A}t}}$$

In this form it is plain that if the value of h is to be recurrent, or the surface movement is restricted to an annual oscillation, the term

$\frac{C}{e^{\frac{0.18\,c}{A}t}}$ must disappear, and the constant of integration must, therefore, be zero. In this case it will be called h_1 where uniformly

$$h_1 = \frac{0.18}{A}\,a\left[\frac{\frac{0.18\,c}{A}\sin t - \cos t}{1+\left(\frac{0.18\,c}{A}\right)^2}\right]$$

Here, also, for the excluded term $(S_1 - Q_1)$ to be zero, the mean supply, S_1, must be constant from year to year and equal to Q_1, a mean discharge, thus fixing the zero from which h is measured at a mean level. As, however, it is well known that the mean supply is not constant from year to year, and that it also passes through a more or less well-defined cycle covering a number of years, this term $(S_1 - Q_1)$ may be again written $(S_{11} - Q_{11}) + (a' \sin t' - c\,h')$, and this multiplied by $(0.18\,dt)$ equals $A\,dh'$, or a second additional element to the surface variation. Here, for all practical purposes, this longer cycle may be assumed as recurrent, with S_{11}, the mean supply, for the number of years equal to Q_{11}, the mean discharge, and the total variation of surface in that case made up of the sum of h_1 and h' measured from the mean level corresponding to Q_{11}.

In this longer cycle, however, the arc t' no longer corresponds to one year, but to a number of years n.

or while $$\frac{T}{t} = \frac{86\,400 \times 365}{2\pi}\,;\quad \frac{T}{t'} = \frac{n \times 86\,400 \times 365}{2\,\pi}$$

and $$t = n\,t',\ \text{and}\ dt = n\,dt'.$$

Hence the differential equation is

$$(a' \sin t' - ch')\ n \times 0.18\,dt' = A\,dh',$$

and its primitive is

$$h' = \frac{n \times 0.18}{A}\,a'\left[\frac{\frac{n \times 0.18\,c}{A}\sin t' - \cos t'}{1+\left(\frac{n \times 0.18\,c}{A}\right)^2}\right] \pm \frac{C}{e^{\frac{n \times 0.18\,c}{A}t'}}\,;$$

in which the term involving the constant of integration now disappears, by taking, as may in general be done, a recurrent variation of h'.

And as the additional oscillation has been taken through a number of years, so also an additional oscillation through a fraction of the year might be taken, and in a sum of these oscillations h may thus

be made to express all the variations of the surface for any such system of regular variations in the monthly and the annual supplies. It is, however, true that the supply in any actual case may be far from regular, and while $(Q_{11} + c\,h)$ always expresses the actual discharge (the difference between the curve and line as noted being insignificant) still $S_{11} + a \sin t + a' \sin t'$, etc., may not so express the actual supply, unless such altogether arbitrary values may be assigned to S_{11} from time to time as will make the expression practically fit any form of irregular variation. It is, therefore, desirable finally to solve Equation I in its most general form with $(S_{11} - Q_{11})$ simply an arbitrary constant, having, of course, alternating + and — values, which at any period of irregularity may be represented by $\triangle S$. In addition to h_1 and h' there is then a third element of variation h'', given by an additional term of the form

$$\frac{0.18}{A} \int \triangle S\, e^{\frac{0.18\,c}{A} t}\, dt$$

or

$$h'' = \frac{\triangle S}{c} \pm \frac{C}{e^{\frac{0.18\,c}{A} t}}$$

And here to determine the general form of the constant of integration C, it is necessary to consider how a lake would rise or fall with such a change in its supply as that of the arbitrary constant $\triangle S$.

Substituting then in Equation II there results

$$(\triangle S - c h'')\, 0.18\, d\,t = A\, d\,h''$$

or

$$d\,t = \frac{A}{0.18\,c} \frac{d\,h''}{\frac{\triangle S}{c} - h''}$$

or

$$t = -\frac{A}{0.18\,c} \log_e \left(\frac{\triangle S}{c} - h''\right) + C$$

And if t be taken as zero where h'' is zero and t_o represents any point of time in such an arbitrary period of t corresponding to the arbitrary value $\triangle S$,

then

$$C = \frac{A}{0.18\,c} \log_e \left(\frac{\triangle S}{c}\right)$$

and

$$t_o = \frac{A}{0.18\,c}\,\log_e\left[\frac{\frac{\triangle S}{c}}{\frac{\triangle S}{c} - h''}\right]$$

or

$$e^{\frac{0.18\,c}{A}t_o} = \frac{\frac{\triangle S}{c}}{\frac{\triangle S}{c} - h''}$$

or

$$h'' = \frac{\triangle S}{c} - \frac{\frac{\triangle S}{c}}{e^{\frac{0.18\,c}{A}t_o}}$$

From which it is seen that a general form for the constant of integration C is $-\frac{\triangle S}{c}$ which, of course, is zero in all the former cases where $\triangle S$ was assumed zero.

The complete equation may be now written as follows:

$$h = h_1 + h' + h'' = \frac{0.18}{A}\,a\left[\frac{\frac{0.18\,c}{A}\sin t - \cos t}{1 + \left(\frac{0.18\,c}{A}\right)^2}\right] + \frac{n \times 0.18}{A}\,a'$$

$$\times\left[\frac{\frac{n \times 0.18\,c}{A}\sin t' - \cos t'}{1 + \left(\frac{n \times 0.18\,c}{A}\right)^2}\right] + \frac{\triangle S}{c}\left[1 - \frac{1}{e^{\frac{0.18c}{A}t_o}}\right] \ldots\ldots \text{(III)}$$

In which t is a continuous angle on a scale in which 360° equal one year, and t_o is the length of a natural arc on the same scale, but taken only through the arbitrary periods in which values may be assigned to $\triangle S$; while again t' is a continuous angle on a scale in which 360° $= n$ years. In this form, in any lake, the function

$$\frac{0.18}{A}\,a\left[\frac{\frac{0.18\,c}{A}\sin t - \cos t}{1 + \left(\frac{0.18\,c}{A}\right)^2}\right]$$ may be computed once for all through

a cycle, and its oscillation then gives at once the ratio of the oscillation of h_1 to a. Or, from the variation of surface in the mean monthly levels the corresponding variation in the supply, expressed by a, is gotten from this simpler proportion. In the same way for h', the var-

iation of supply from year to year expressed by a' may be brought into a like proportion with the variation in the mean annual levels. While finally for given periods of t_o corresponding proportions may be made up that express immediately the $\triangle S$ for any irregularity.

The relation of the variation of level in any lake to its variable supply may be here left for the more general problem which includes not only this, but its resultant effect on a lake below, and so on through a chain of lakes; and not to needlessly extend the problem, it is desirable in this case to use the simplest possible form for h. It is plain that in so far as h_1 and h' are concerned, it is only necessary to use the term h_1, for what will be generally true of the short cycle will be equally true of the long, and all equations deduced from the first can be immediately converted into equations for the second by substituting in the place of (0.18) the value of (n 0.18) throughout, remembering, of course, that degrees in the angles t' are also to be taken as covering n times as many days as they do in the angles t; but the wholly different character of the term for h'' makes it necessary to exclude it altogether or to consider its effects down entirely as a question by itself.

This last has in fact been done, but it is all so much a matter of the particular case, that it will not be included in this study. Indeed, from the extreme where $\triangle S$ is assumed a permanent change in the mean supply, it is evident at once that in time any number of lakes would all come to about the same difference in level, represented by $\frac{\triangle S}{c}$; though it may be noted that none could absolutely reach this level until t_o became infinite. And while in very long periods there are practically no differences in the effects of this function on lakes below, at the same time in very short periods it has practically no effect whatever on them, so that in the range of ordinary irregularities in supply, it may simply be taken as rapidly disappearing in its effects on the lower lakes, but exactly how rapidly would depend on the length of the irregularity. It is only perhaps in the event of a system of storage in an upper lake through one season to be let out at another that the subject would have any general interest and there its effects are best worked out as a special case.

The more general problem of the effects of variable supplies on the levels of a chain of lakes is therefore confined to the consideration of the recurrent yearly oscillations h_1 and its effects below; and the

following general system of notation is taken for the resulting surface movements in a chain of n lakes.

$$I\,h_1$$
$$I\,h_2 = II\,h_2$$
$$I\,h_3 = II\,h_3 = III\,h_3$$
$$I\,h_4 = II\,h_4 = III\,h_4 = IV\,h_4$$
$$I\,h_5 = II\,h_5 = III\,h_5 = IV\,h_5 = V\,h_5$$
etc., etc., etc., etc., etc.,
$$I\,h_n = II\,h_n = III\,h_n = IV\,h_n = V\,h_n \text{ etc. } [n]\,h_n.$$

Here the vertical lines represent the effects of the primary supply in the basin of each respective lake all the way down, and the horizontal represent all the effects that as a whole make up the surface movement of any lake in the system. In correspondence with this the areas of the respective lakes will be represented by A_1, A_2, etc., A_n, the "a" in the variations of supply $a \sin t$ by a_1, a_2, etc., a_n, and the change of discharge per foot of rise or fall in the outlets by C_1, C_2, etc., C_n.

Considering, now, the supply in the basin of the first lake and its effects down, there results:

$$I\,h_1 = a_1 \times \frac{0.18}{A_1} \times \frac{1}{1 + \left(\frac{0.18\,c_1}{A_1}\right)^2} \left[\frac{0.18\,c_1}{A_1} \sin t - \cos t\right]$$

and as the factors $\frac{0.18}{A}$ and $\frac{1}{1 + \left(\frac{0\,18\,c}{A}\right)^2}$ will recur continually in these operations, it is convenient to represent them by the more simple terms K and V, writing them for their respective values of A and C, as K_1, K_2; V_1, V_2, etc.

With this substitution

$$I\,h_1 = a_1\,K_1\,V_1\,(K_1\,c_1 \sin t - \cos t)$$

and as for the second lake $c_1 \times h_1$ is the resulting variation of supply and $c_2 \times h_2$, the corresponding variation in its discharge, from equation II there results:

$$[a_1\,c_1\,K_1\,V_1\,(K_1\,c_1 \sin t - \cos t) - c_2\,h_2]\;0.18\,d\,t = A_2\,d\,h_2$$

or

$$d\,h_2 + \frac{0.18\,c_2}{A_2}\,h_2\,d\,t = \frac{0.18}{A_2}\,a_1\,c_1\,K_1\,V_1\,(K_1\,c_1 \sin t - \cos t)\,d\,t$$

or

$$h_2\,e^{K_2 c_2 t} = a_1\,c_1\,K_1\,K_2\,V_1 \int (K_1\,c_1 \sin t - \cos t)\,e^{K_2 c_2 t}\,d\,t.$$

And in general, following the process of the former integration by parts,

$$\int e^{Kct} \sin t\, dt = \frac{Kc \sin t - \cos t}{1+(Kc)^2}\ e^{Kct}$$

and

$$\int e^{Kct} \cos t\, dt = \frac{Kc \cos t + \sin t}{1+(Kc)^2}\ e^{Kct}$$

so that

$$h_2 e^{K_2 c_2 t} = a_1 c_1 K_1 K_2 V_1 V_2 \,|\, K_1 c_1 (K_2 c_2 \sin t - \cos t) - (K_2 c_2 \cos t + \sin t)]\ e^{K_2 c_2 t}$$

or

$$I h_2 = a_1 c_1 K_1 K_2 V_1 V_2 [(K_2 c_2 K_1 c_1 - 1) \sin t - (K_2 c_2 + K_1 c_1) \cos t].$$

And again for the third lake with $c_2 \times h_2$ supply, and $c_3 \times h_3$ discharge, $h_3 e^{K_3 c_3 t} = a_1 c_1 c_2 K_1 K_2 K_3 V_1 V_2 \int [(K_2 c_2 K_1 c_1 - 1) \sin t - (K_2 c_2 + K_1 c_1) \cos t]\ e^{K_3 c_3 t} dt = a_1 c_1 c_2 K_1 K_2 K_3 V_1 V_2 V_3 [(K_2 c_2 K_1 c_1 - 1)(K_3 c_3 \sin t - \cos t) - (K_2 c_2 + K_1 c_1)(K_3 c_3 \cos t + \sin t)]\ e^{K_3 c_3 t}$

or $I h_3 =$

$$a_1 c_1 c_2 K_1 K_2 K_3 V_1 V_2 V_3 \Big[|K_3 c_3 (K_2 c_2 K_1 c_1 - 1) - (K_2 c_2 + K_1 c_1)] \sin t - [K_3 c_3 (K_2 c_2 + K_1 c_1) + (K_2 c_2 K_1 c_1 - 1)] \cos t \Big]$$

and, in general, calling*

$$I h_n = a_1 \{c_1\}^{n-1} \{K_1\}^n \{V_1\}^n [I \alpha_n \sin t + I \beta_n \cos t] \ldots\ldots (IV)$$

in which α and β are formed as follows:

	$I\alpha =$	$I\beta =$
$I h_1 = (K_1 c_1)$	....	$= -1$
$I h_2 = (K_2 c_2 K_1 c_1 - 1)$	.....	$= -(K_2 c_2 + K_1 c_1)$
$I h_3 = K_3 c_3 (K_2 c_2 K_1 c_1 - 1) - (K_2 c_2 + K_1 c_1).$		$= -[K_3 c_3 (K_2 c_2 + K_1 c_1) + (K_2 c_2 K_1 c_1 - 1)]$

and finally

$$I h_n = K_n c_n \alpha_{n-1} + \beta_{n-1} \ldots\ldots = K_n c_n \beta_{n-1} - \alpha_{n-1}.$$

In the case of $II h_2$ begin with

$$II h_2 = a_2 K_2 V_2 [K_2 c_2 \sin t - \cos t]$$

and in the same way finally in the n^{th} lake there results

$$II h_n = a_2 \{c_2\}^{n-1} \{K_2^n\} \{V_2^n\} [II \alpha_n \sin t + II \beta_n \cos t]$$

and so on from lake to lake.

From these equations the effect of one lake upon another, and the total surface movements in any lake, are easily computed. Thus, to

* In this paper the braces $\{\ \}$ are used exclusively as a "factorial sign."

illustrate, the case of a chain of five lakes may be taken, each with a surface area of 10 000 square miles, with a 100 000 cu. ft. per second variation in the supply, entering each lake as a simultaneous oscillation, and with outlet capacities downward of, respectively, 10, 15, 20, 25 and 30 thousands of cubic feet per second per foot of rise or fall. Then

$$A_1 = A_2 = A_3 = A_4 = A_5 = 10\,000$$

$$a_1 = a_2 = a_3 = a_4 = a_5 = 50\,000$$

$C_1 = 10\,000$; $C_2 = 15\,000$; $C_3 = 20\,000$; $C_4 = 25\,000$; $C_5 = 30\,000$.

Hence

$K C_1 = 0.18$; $K C_2 = 0.27$; $K C_3 = 0.36$; $K C_4 = 0.45$; $K C_5 = 0.54$

and

$I\alpha_1 = +0.18$
$\quad 0.27$
$\quad 126$
$\quad 036$
$\quad +0.0486$
$\quad -1.0000$

$I\alpha_2 = -0.9514$
$\quad 0.36$
$\quad 57084$
$\quad 28542$
$\quad -0.3425$
$\quad -0.4500$

$I\alpha_3 = -0.7925$
$\quad 0.45$
$\quad 39625$
$\quad 31700$
$\quad -0.3566$
$\quad +0.7894$

$I\alpha_4 = +0.4328$
$\quad 0.54$
$\quad 17312$
$\quad 21640$
$\quad +0.2337$
$\quad +1.1477$

$I\alpha_5 = +1.3814$

$I\beta_1 = -1.0$
$\quad 0.27$
$\quad -0.27$
$\quad -0.18$

$I\beta_2 = -0.45$
$\quad 0.36$
$\quad 270$
$\quad 135$
$\quad -0.1620$
$\quad +0.9514$

$I\beta_3 = +0.7894$
$\quad 0.45$
$\quad 39470$
$\quad 31576$
$\quad +0.35523$
$\quad +0.7925$

$I\beta_4 = +1.1477$
$\quad 0.54$
$\quad 45908$
$\quad 57385$
$\quad +0.6198$
$\quad -0.4328$

$I\beta_5 = +0.1870$

and so for $II\,\alpha_2$, $II\,\beta_2$, to $II\,\alpha_5$, $II\,\beta_5$, etc., giving:

α_1	α_2	α_3	α_4	α_5	
+	−	−	+	+	
0.1800	0.9514	0.7925	0.4328	1.3814	*I*
	+	−	−	+	
	0.2700	0.9028	1.0363	0.0597	*II*
		+	−	−	
		0.3600	0.8380	1.2625	*III*
			+	−	
			0.4500	0.7570	*IV*
				+	
				0.5400	*V*

β_1	β_2	β_3	β_4	β_5	
−	−	+	+	+	
1.0000	0.4500	0.7894	1.1477	0.1870	*I*
	−	−	+	+	
	1.0000	0.6300	0.6193	1.3707	*II*
		−	−	+	
		1.0000	0.8100	0.4006	*III*
			−	−	
			1.0000	0.9900	*IV*
				−	
				1.0000	*V*

Again, the following are the logs of the general factor $a\,\{c\}^{n-1}\,\{K\,V\}^n$ which are easily made up by repeated additions:

$\{\ \}^1$	$\{\ \}^2$	$\{\ \}^3$	$\{\ \}^4$	$\{\ \}^5$	
−	−	−	−	−	
1.940394	1.165108	2.543547	2.019764	3.561849	*I*
	−	−	−	−	
	1.923683	1.302122	2.778940	2.321024	*II*
		−	−	−	
		1.901318	1.377535	2.919620	*III*
			−	−	
			1.874157	1 416242	*IV*
				−	
				1.843114	*V*

and multiplying here the respective values of α and β by the corresponding general factors and representing the products by $I\,\{\alpha_1\}$, $I\,\{\beta_1\}$, &c., there results finally:

$\{\alpha_1\}$	$\{\alpha_2\}$	$\{\alpha_3\}$	$\{\alpha_4\}$	$\{\alpha_5\}$	
+	−	−	+	+	
0.15692	0.13915	0.02770	0.00453	0.00504	*I*
	+	−	−	+	
	0.22649	0.18101	0.06229	0.00125	*II*
		+	−	−	
		0.28683	0 19988	0 10492	*III*
			+	−	
			0 33680	0.19740	*IV*
				+	
				0.37628	*V*

$\{\beta_1\}$	$\{\beta_2\}$	$\{\beta_3\}$	$\{\beta_4\}$	$\{\beta_5\}$	
—	—	+	+	+	
0.87176	0.06581	0.02760	0.01201	0.00068	*I*
	—	—	+	+	
	0.83885	0.12632	0.03723	0.02871	*II*
		—	—	+	
		0.79674	0.19321	0.03329	*III*
			—	—	
			0 74844	0.25815	*IV*
				—	
				0.69681	*V*

Taking now t at intervals of 30° corresponding with the twelfth of the year and practically with the monthly means:

t	sin t	cos t
0°	0	+1
30°	+0.5	+0.866
60°	+0.866	+0.5
90°	+1	0
120°	+0.866	−0.5
150°	+0.5	−0.866
180°	0	−1

And with these values, from the equation $h = \{\alpha\} \sin t + \{\beta\} \cos t$, the whole cycle of each of these variations is readily made up as follows:

$t =$	0°	30°	60°	90°	120°	150°	180°
$I \{\alpha_1\} \sin t =$	+0.0000	+0.0785	+0.1359	+0.1569	+0.1359	+0.0785	+0.0000
$I \{\beta_1\} \cos t =$	−0.8718	−0.7550	−0.4359	−0.0000	+0.4359	+0.7550	+0.8718
$I h_1 =$	−0.8718	−0.6765	−0.3000	+0.1569	+0.5718	+0.8335	+0.8718
and with signs reversed	$t =$ 210°	240°	270°	300°	330°	360°	

completing the cycle.

These values for the half cycle are here given in series from the primary supply on, to its effect in the last lake.

	0°	30°	60°	90°	120°	150°	180°
$I h_1$	−0.8718	−0.6765	−0.3000	+0.1569	+0.5718	+0.8335	+0.8718
$I h_2$	−0.0658	−0.1266	−0.1534	−0.1391	−0.0876	−0.0126	+0.0658
$I h_3$	+0.0276	+0.0101	−0.0102	−0.0277	−0.0378	−0.0377	−0.0276
$I h_4$	+0.0120	+0.0127	+0.0099	+0.0045	−0.0021	−0.0061	−0.0120
$I h_5$	+0.0007	+0.0031	+0.0047	+0.0050	+0.0041	+0.0019	−0.0007
$II h_2$	−0.8388	−0.6133	−0.2233	+0.2265	+0.6155	+0.8397	+0.8388
$II h_3$	−0.1263	−0.1999	−0.2200	−0.1810	−0.0936	+0.0189	+0.1263
$II h_4$	+0.0372	−0.0011	−0.0353	−0.0622	−0.0725	−0.0633	−0.0372
$II h_5$	+0.0287	+0.0255	+0.0155	+0.0012	−0.0133	−0.0243	−0.0287
$III h_3$	−0.7967	−0.5466	−0.1500	+0.2868	+0.6468	+0.8334	+0.7967
$III h_4$	−0.1932	−0.2672	−0.2697	−0.1999	−0.0765	+0.0674	+0.1932
$III h_5$	+0.0333	−0.0237	−0.0743	−0.1049	−0.1075	−0.0813	−0.0333
$IV h_4$	−0.7484	−0.4798	−0.0825	+0.3368	+0.6659	+0.8166	+0.7484
$IV h_5$	−0.2581	−0.3223	−0.3000	−0.1974	−0.0418	−0.1249	+0.2581
$V h_5$	−0.6968	−0.4154	−0.0225	+0.3763	+0.6743	+0.7916	+0.6968

And these values summed algebraically for their respective lakes would give in each case the total of the surface movement there, but it is easier to sum at once the coefficients of sin t and cos t and calculate directly the cycles of these resultant oscillations.

Giving the following values:

		0°	30°	60°	90°	120°	150°	180°
I	h_1	−0.8718	−0.6765	−0.3000	+0.1569	+0.5718	+0.8335	+0.8718
I to II	h_2	−0.9047	−0.7398	−0.3767	+0.0873	+0.5279	+0.8272	+0.9047
I to III	h_3	−0.8955	−0.7364	−0.3801	+0.0781	+0.5153	+0.8146	+0.8955
I to IV	h_4	−0.8924	−0.7332	−0.3777	+0.0792	+0.5147	+0.8124	+0.8924
I to V	h_5	−0.8922	−0.7326	−0.3766	+0.0802	+0.5156	+0.8128	+0.8922

These values are shown graphically in Fig. 1, and while they show the general form of the oscillations, it is very evident that they do not give, with any degree of accuracy, the extremes. It is seen, of course, that while $a \sin t$, or the supply, is in all cases at its maximum at 90°, its corresponding rise reaches its maximum in the first lake somewhere between the 150° and 180° periods, and that this again as a supply is once more retarded in the rise of the second lake, and so on from lake to lake; but the periods and the values of these maxima require additional calculations.

The value of t at these points of maxima or minima is, of course, determined by the condition $\frac{dh}{dt} = 0$, and in the general form of the equation

$$h_n = a \left\{c\right\}^{n-1} \left\{K\right\}^n \left\{V\right\}^n \left[\alpha_n \sin t + \beta_n \cos t\right]$$

$$\frac{dh_n}{dt} = 0, \text{ gives } \alpha_n \cos t = \beta_n \sin t, \text{ or } \tan t = \frac{\alpha}{\beta}$$

The two angles in the cycle differing by 180°, that correspond to this value of the tan t, are, of course, the points of maximum and minimum respectively; and by substituting either in the equation for h the + or − extreme is given. It is, however, enough simply to consider the maximum, as before only the half cycle was calculated, every value of h having, of course, a corresponding value with an opposite sign after a period of 180°.

As the effects of a primary supply are traced down, the retardation throws the maximum into later and later quadrants, and these must be followed by the successive change of signs in the term $\frac{\alpha}{\beta}$.

Thus, in the case of I, the original supply, $a \sin t$, has its maximum as noted at 90°, while for I h_1, $\frac{\alpha_1}{\beta_1}$ is minus, and is therefore in the

second quadrant, and its angle 10° 12′ gives the absolute position as 180° 00′ — 10° 12′ = 169° 48′, and substituting this angle for t gives the value of 0.8858 for the maximum of $I\,h_1$. Again for $I\,h_2$, $\frac{\alpha_2}{\beta_2}$ is plus, and is thus in the third quadrant, and its angle 64° 41′ gives its position as 180° 00′ + 64° 41′ = 244° 41′, and its value 0.1539. In the same way $\frac{\alpha_3}{\beta_3}$ is minus and lies in the fourth quadrant, and its angle 45° 07′ places it at 360° 00′ — 45° 07′ = 314° 53′ with the maximum value 0.0391. While finally $\frac{\alpha_4}{\beta_4}$ and $\frac{\alpha_5}{\beta_5}$ are both plus and lie in the fifth quadrant at the points respectively 360° 00′ + 20° 40′ = 380° 40′ and 360° 00′ + 82° 18′ = 442° 18′, and maxima of 0.0128 and 0.0051.

The positions of these maxima and their values through this whole series of oscillations are tabulated as follows:

Angle t of maximum.

h_1	h_2	h_3	h_4	h_5	
169° 48′	244° 41′	314° 53′	380° 40′	442° 18′	*I*
	164° 53′	235° 05′	300° 52′	362° 30′	*II*
		160° 12′	225° 58′	287° 36′	*III*
			155° 46′	217° 24′	*IV*
				151° 38′	*V*

Value of maximum h.

h_1	h_2	h_3	h_4	h_5	
0.8858	0.1539	0.0391	0.0128	0.0051	*I*
	0.8689	0.2207	0.0726	0.0287	*II*
		0.8468	0.2780	0.1101	*III*
			0.8207	0.3250	*IV*
				0.7919	*V*

From these values of t it is seen at once that the difference between the maxima of $I\,h_1$ and $I\,h_2$, or 244° 41′ — 169° 48′ = 74° 53′ is also the difference between 164° 53′ and 90°, or between the maximum of supply, $a \sin t$, and its rise, $II\,h_2$. And, again, 70° 12′ is the general difference between h_2 and h_3; 65° 46′ + between h_3 and h_4, and 61° 38′ between h_4 and h_5; which are also in each case the difference between 90°, the maxima of the respective supplies, $a \sin t$, and their corresponding oscillations in the first lake. It is then plain that the retardation in any lake is a constant dependent upon the physical proper-

ties of that lake, and in no way upon the magnitude of the oscillations passing through it. And just as it has been seen that any oscillation passing from a given lake to the next is retarded by a fixed time; so it may be seen from the corresponding maximum values of h that they are also reduced by a fixed percentage.

It is time therefore to turn from the series of equations which have just been considered, to determine the formulas for this retardation and reduction from lake to lake; for it is very evident that they are a much more general expression for the laws of lake movements than equations which simply give direct values for assumed times and supplies.

First, for the reduction ratio or percentage, which will be called p; noting that it is enough to express it in the case of *I* from h_1 to h_n, for it is also the same from lake to lake in the case of *II*, *III*, etc.; and omitting, therefore, the *I* in the general equation, and representing by h' and t' respectively the maximum rise and the time corresponding to it, there results:

$$h'_n = a \left\{c_1\right\}^{n-1} \left\{K_1\right\}^n \left\{V_1\right\}^n [\alpha_n \sin t' + \beta_n \cos t'] \ldots\ldots\ldots (1)$$

and

$$\frac{\alpha_n}{\beta_n} = \frac{\sin t'}{\cos t'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

or from (2)

$$\sin t' = \frac{\alpha_n}{\sqrt{\alpha_n^2 + \beta_n^2}} \text{ and } \cos t' = \frac{\beta_n}{\sqrt{\alpha_n^2 + \beta_n^2}}$$

and eliminating t' between equations (1) and (2)

$$h'_n = a \left\{c_1\right\}^{n-1} \quad K_1\Big\}^n \left\{V_1\right\}^n \sqrt{\alpha_n^2 + \beta_n^2}.$$

Again from the law of formation in the general values of α and β, there results:

$\alpha_1 = K_1 c_1$ or $\alpha_1^2 = \overline{K_1 c_1}^2$. $\beta_1 = -1$; $\beta_1^2 = 1$. Hence, $\alpha_1^2 + \beta_1^2 = 1 + \overline{K_1 c_1}^2$

And again:

$$\alpha_2 = K_2 c_2 \alpha_1 + \beta_1 \text{ or } \alpha_2^2 = \overline{K_2 c_2}^2 \alpha_1 + 2 K_2 c_2 \alpha^2_1 \beta_1 + \beta_1^2$$

$$\beta_2 = K_2 c_2 \beta_1 - \alpha_1 \text{ or } \beta_2^2 = \overline{K_2 c_2}^2 \beta_1^2 - 2 K_2 c_2 \alpha_1 \beta_1 + \alpha_1^2$$

$$\text{Hence, } \alpha_2^2 + \beta_2^2 = \overline{K_2 c_2}^2 \left(\alpha_1^2 + \beta_1^2\right) + \alpha_1^2 + \beta_1^2$$

or

$$\alpha_2^2+\beta_2^2=\left(\alpha_1^2+\beta_1^2\right)\left(1+\overline{K_2c_2}^2\right)=\left(1+\overline{K_1c_1}^2\right)\left(1+\overline{K_2c_2}^2\right)$$

and so on, until in general

$$\alpha_n^2+\beta_n^2=(1+\overline{K_1c_1}^2)\,(1+\overline{K_2c_2}^2)\,(1+\overline{K_3c_3}^2)\;\ldots\;(1+\overline{K_nc_n}^2)=\left\{1+\overline{K_1c_1}^2\right\}^n$$

from which finally there is obtained—

$$\frac{h'_n}{h'_{n-1}}=\frac{a\left\{c_1\right\}^{n-1}\left\{K_1\right\}^n\left\{V_1\right\}^n\sqrt{\left\{1+\overline{K_1c_1}^2\right\}^n}}{a\left\{c_1\right\}^{n-2}\left\{K_1\right\}^{n-1}\left\{V_1\right\}^{n-1}\sqrt{\left\{1+\overline{K_1c_1}^2\right\}^{n-1}}}=p_n$$

Or

$$p_n=c_{n-1}\,K_n\,V_n\sqrt{1+\overline{K_nc_n}^2}$$

And writing now for K_n and V_n their respective values:

$$K_n=\frac{0.18}{A_n}$$

$$V_n=\frac{1}{1+\left(\frac{0.18\,c_n}{A_n}\right)^2}$$

$$p_n=\frac{0.18\,c_{n-1}}{A_n}\,\frac{\sqrt{1+\left(\frac{0.18\,c_n}{A_n}\right)^2}}{1+\left(\frac{0.18\,c_n}{A_n}\right)^2}=\frac{\frac{0.18\,c_{n-1}}{A_n}}{\sqrt{1+\left(\frac{0.18c_n}{A_n}\right)^2}}\quad\ldots\ldots(V)$$

A value depending simply upon the area of the n^{th} lake and the discharging capacity of its inlet and its outlet.

Again, taking also the case *I* for the retardation, which will be called R_i and representing by t'_{n-1} and t'_n the time of the maxima in the $n-1$ and the n^{th} lake, respectively, there results:

$$\sin R_n=\sin\left(t'_n-t'_{n-1}\right)=\sin t'_n\cos t'_{n-1}-\cos t'_n\sin t'_{n-1}$$

and as before

$$\sin t_n{}^1=\frac{\alpha_n}{\sqrt{\alpha_n{}^2+\beta_n{}^2}}=\frac{\alpha_n}{\sqrt{\left\{1+\overline{K_1c_1}^2\right\}^n}}\text{ and }\cos t'_n=\frac{\beta_n}{\sqrt{\alpha_n{}^2+\beta_n{}^2}}=\frac{\beta_n}{\sqrt{\left\{1+\overline{K_1c_1}^2\right\}^n}}$$

Hence,

$$\sin R_n=\frac{\alpha_n\,\beta_{n-1}}{\left\{1+\overline{K_1c_1}^2\right\}^{n-1}\sqrt{1+\overline{K_nc_n}^2}}-\frac{\alpha_{n-1}\,\beta_n}{\left\{1+\overline{K_1c_1}^2\right\}^{n-1}\sqrt{1+\overline{K_nc_n}^2}}$$

and as $\alpha_n\,\beta_{n-1}=(K_n\,c_n\,\alpha_{n-1}+\beta_{n-1})\,\beta_{n-1}$

and $$\alpha_{n-1}\,\beta_n = (K_n\,c_n\,\beta_{n-1} - \alpha_{n-1})\,\alpha_{n-1}$$

$$\alpha_n\,\beta_{n-1} - \alpha_{n-1}\,\beta_n = \alpha^2_{n-1} + \beta^2_{n-1} = \left\{1 + \overline{K_1\,c_1}^{\,2}\right\}^{n-1}$$

therefore,

$$\sin R_n = \frac{\left\{1 + \overline{K_1\,c_1}^{\,2}\right\}^{n-1}}{\left\{1 + \overline{K_1\,c_1}^{\,2}\right\}^{n-1}\sqrt{1 + \overline{K_n\,c_n}^{\,2}}} = \frac{1}{\sqrt{1 + \overline{K_n\,c_n}^{\,2}}} =$$

$$\frac{1}{\sqrt{1 + \left(\dfrac{0.18\,c_n}{A_n}\right)^2}} \qquad \text{(VI)}$$

From equation (VI) it is seen that in passing through a given lake, oscillations of supply are retarded by this period, which depends alone upon the area and the outlet of that lake; but on comparing it with equation (V) it is evident at once that p_n is not a reduction of the same general character. It gives the ratio $\dfrac{h'_n}{h'_{n-1}}$, and is a most convenient function for calculating the effects of a given oscillation in any lake upon all the lakes that follow it; for at any point $h'_{n-1} \times p_n = h'_n$; but with exactly the same oscillation of supply in the n^{th} lake, and, in consequence, the same value of h'_n, p_n may have any number of values, as the oscillation from above which gives this supply has different outlets.

Thus, finally, there may be considered the reduction effected by a given lake on the variations of supply; or a reduction of the same general character as the retardation; and calling $\triangle S'_n$ and $\triangle Q'_n$ the maxima for the oscillations of supply and discharge respectively in the n^{th} lake, this absolute percentage of reduction, which will be called P, is given by the ratio $\dfrac{\triangle Q'_n}{\triangle S'_n}$.

This is easily determined from equation (V),

for $$h'_n c_n = \triangle Q'_n \text{ and } h'_{n-1}\,c_{n-1} = \triangle S'_n$$

or $$\frac{\triangle Q'_n}{\triangle S'_n} = \frac{h'_n}{h'_{n-1}} \times \frac{c_n}{c_{n-1}}$$

and as $$p_n = \frac{h'_n}{h'_{n-1}} = \frac{\dfrac{0.18\,c_{n-1}}{A_n}}{\sqrt{1 + \left(\dfrac{0.18\,c_n}{A_n}\right)^2}}$$

there results

$$p_n = \frac{c_n}{c_{n-1}} \times \frac{\frac{0.18\ c_{n-1}}{A_n}}{\sqrt{1+\left(\frac{0.18\ c_n}{A_n}\right)^2}} = \frac{\frac{0.18\ c_n}{A_n}}{\sqrt{1+\left(\frac{0.18\ c_n}{A_n}\right)^2}} \quad \ldots\ldots\ldots (VII)$$

Thus in equations (*VI*) and (*VII*) may be seen finally, from the area and the outlet of the lake itself, its whole effect upon any oscillations of supply. It simply retards it by this given time, and reduces it to this given percentage.

Indeed, these last equations might have been originally determined simply from the consideration of the general relations between an oscillation of supply, $a \sin t$, and its corresponding oscillation of level.

Or as before

$$h = a\ \frac{\frac{0.18}{A}}{1+\left(\frac{0.18\ c}{A}\right)^2}\left[\frac{0.18\ c}{A}\sin t - \cos t\right]$$

and again taking t' as the angle corresponding to the maximum h' there results from $\frac{dh}{dt} = 0$

$$-\frac{0.18\ c}{A}\cos t' = \sin t'; \text{ and } \sin t' = \frac{\frac{0.18\ c}{A}}{\sqrt{1+\left(\frac{0.18\ c}{A}\right)^2}}$$

and

$$h \sin t' = a\ \frac{\frac{0.18}{A}}{1+\left(\frac{0.18\ c}{A}\right)^2}\left[\frac{0.18\ c}{A}(\sin t \sin t' + \cos t \cos t')\right]$$

or

$$h \times \frac{\frac{0.18\ c}{A}}{\sqrt{1+\left(\frac{0.18\ c}{A}\right)^2}} = a \times \frac{\frac{0.18}{A}}{1+\left(\frac{0.18\ c}{A}\right)^2} \times \frac{0.18\ c}{A}\cos(t - t')$$

and finally

$$h = \frac{\frac{0.18}{A}}{\sqrt{1+\left(\frac{0.18\ c}{A}\right)}}\ a \sin(90^\circ + t - t') \quad \ldots\ldots\ldots\ldots\ldots (VIII)$$

Which, simply reduced by the factor $\frac{\frac{0.18}{A}}{\sqrt{1+\left(\frac{0.18\ c}{A}\right)^2}}$, is alto-

gether a similar oscillation to $a \sin t$, only shifted to a new origin of time, with a zero of 90° before the maximum t', or zero where $t' - 90° = t$, as the origin in the original, $a \sin t$, was zero 90° before its maximum; and as this is true of any oscillation in any lake, it is just as applicable to the n^{th} oscillation as the first; and the formulas for reduction and retardation determined from it apply to all.

From equation (*VIII*) it is very evident that the maximum of level is given in all cases by the equation

$$h' = \frac{\frac{0.18}{A}}{\sqrt{1 + \left(\frac{0.18\, c}{A}\right)^2}}\, a,$$

where a represents the maximum of any supply oscillation from any source.

And writing f for this function $\dfrac{\frac{0.18}{A}}{\sqrt{1 + \left(\frac{0.18\, c}{A}\right)^2}}$,

and for a given lake in a series f_n, there results $\dfrac{h'_n}{f_n} = a_n$, by which, from the observed maximum of rise h'_n in any case, the corresponding supply may be at once determined; while, again, this is easily carried down through any number of lakes by their successive values of the retardation and the reduction ratio.

An analysis of the mean oscillations of the Great Lakes is here given to illustrate the application of the above formulas to the study of such data.

In this case the following data are given :

	A.	*C.*	*Log f.*	*Log p.*	*t'.*	*R.*
I. Superior	31 800	17 000	$\bar{6}.7505$	.....	174° 30′	84 30′
II. Michigan-Huron	45 600	26 000	$\bar{6}.5944$	$\bar{2}.8245$	174° 06′	84° 06′
III. St. Clair	495	26 000	$\bar{5}.5826$	$\bar{1}.9976$	96° 02′	6° 02′
IV. Erie	10 000	30 000	$\bar{5}.1995$	$\bar{1}.6147$	151° 38′	61° 38′
V. Ontario	7 450	34 000	$\bar{5}.2711$	$\bar{1}.7482$	140° 36′	50° 36′

The means of twenty-five years, from 1871 to 1895, for Superior are as follows:

Observed oscillations.				Computed trial oscillations.		
				t	Ih_1	t
September.	+0.47	+0.47	September...	174°	+0.500	174°
				− 30°		+ 30°
August.....	+0.45	+0.44	October.....	144°	+0.433	204°
				− 30°		+ 30°
July.....	+0.37	+0.29	November...	114°	+0.250	234°
				− 30°		+ 30°
June.....	+0.16	+0.01	December...	84°	0.000	264°
				− 30°		+ 30°
May.....	−0.15	−0.25	January.....	54°	−0.250	294°
				− 30°		+ 30°
April.....	−0.45	−0.40	February....	24°	−0.433	324°
				− 30°		+ 30°
March.....	−0.46	−0.46	March.....	− 06°	−0.500	354°

By plotting and comparing this trial oscillation with the observed one, it is seen that the zero of its t corresponds closely with the mean March period, and that its range fits the whole series of observed values about as well as possible.

The value of the maximum of Ih_1 is therefore taken as 0.50, and the oscillation of its supply is therefore—

$$\left.\begin{array}{rl} & \log \\ 0.50 = & \overline{1}.6990 \\ f_1 = & \overline{6}.7505 \\ a_1 = & 4.9495 \quad = 83\ 480. \end{array}\right\} \text{ or } IS = 83\ 480 \sin t$$

The effects of this upon all the lakes below is readily carried down as follows:

MAXIMUM EFFECTS.

			log.
It'_1	= 174° 30′	Ih'_1	= $\overline{1}.6990$ = 0.500
$+ R_2$	84° 08′	$+ \log p_2$	$\overline{2}.8245$
It'_2	= 258° 38′	Ih'_2	= $\overline{2}.5235$ = 0.033
$+ R_3$	6° 02′	$+ \log p_3$	$\overline{1}.9976$
It'_3	= 264° 40′	Ih'_3	= $\overline{2}.5211$ = 0.033
$+ R_4$	61° 38′	$+ \log p_4$	$\overline{1}.6147$
It'_4	= 326° 18′	Ih'_4	= $\overline{2}.1358$ = 0.014
$+ R_5$	50° 36′	$+ \log p_5$	$\overline{1}.7482$
It'_5	= 376° 54′	Ih'_5	= $\overline{3}.8840$ = 0.008

Again, from these times and values of the maxima, the complete oscillations are as follows:

Michigan-Huron.		St. Clair.		Erie.		Ontario.	
t.	Ih_2.	t.	Ih_3.	t.	Ih_4.	t.	Ih_5.
259°	+0.033	265°	+0.033	326°	+0.014	377°	+0.008
− 30°		− 30°		− 30°		− 30°	
229°	+0.029	235°	+0.029	296°	+0.012	347°	+0.007
− 30°		− 30°		− 30°		− 30°	
199°	+0.017	205°	+0.017	266°	+0.007	317°	+0.004
− 30°		− 30°		− 30°		− 30°	
169°	+0.000	175°	0.000	236°	0.000	287°	0.000
− 30°		− 30°		− 30°		− 30°	
139°	−0.017	145°	−0.017	206°	−0.007	257°	−0.004
− 30°		− 30°		− 30°		− 30°	
109°	−0.029	115°	−0.029	176°	−0.012	227°	−0.007
− 30°		− 30°		− 30°		− 30°	
79°	−0.033	− 85°	−0.033	146°	−0.014	197°	−0.008

Again, from the monthly means of the Michigan-Huron data, the effect there of the oscillation from Superior is first subtracted, and the result gives the oscillation of Michigan-Huron from the supply of its own water-shed. This is called the local observed oscillation, and with its corresponding computed trial oscillation is given as follows:

Local observed oscillation.				Computed trial oscillation.		
				t.	IIh_2	t.
July.....	+0.52	+0.52	July.....	174°	+0.520	174°
June.....	+0.49	+0.46	August.....	144°	+0.450	204°
May.....	+0.23	+0.22	September.	114°	+0.260	234°
April.....	−0.03	−0.03	October....	84°	0.000	264°
March.....	−0.27	−0.29	November..	54°	−0.260	294°
February....	−0.42	−0.53	December..	24°	−0.450	324°
January.....	−0.48	−0.48	January...	− 06°	−0.520	354°

Plotting and comparing these oscillations, it is seen that the maximum of 0.52 is best fitted to the whole series, from which—

$$\left.\begin{aligned} \log 0.52 &= \overline{1}.7160 \\ f_2 &= \overline{6}.5944 \\ a_2 &= \overline{5}.1216 = 129\,470 \end{aligned}\right\} \quad \text{or } II\,S = 129\,470 \sin t.$$

The zero of t, however, must in this case be put some 4° later than the mid-January period; and to bring here the computed values of this oscillation and its effects below to the same time scale as Superior, 56°

must in all cases be subtracted from its values of t. This, however, may be done at once on the retardations, and then the Michigan-Huron effects on the Superior time scale are as follows:

MAXIMUM EFFECTS.

Michigan-Huron time.			log
	$II\,t'_2$	$= 174° 08'$	
		$- 56°$	
Superior time.	$II\,t'_3$	$= 118° 08'$	$II\,h'_2$ $\bar{1}.7160 = 0.520$
	$+ R_3$	$6° 02'$	$+ \log p_3$ $\bar{1}.9976$
	$II\,t'_3$	$= 124° 10'$	$II\,h'_3$ $= \bar{1}.7136 = 0.517$
	$+ R_4$	$61° 38'$	$+ \log p_4$ $\bar{1}.6147$
	$II\,t'_4$	$= 185° 48'$	$II\,h'_4$ $= \bar{1}.3283 = 0.213$
	$+ R_5$	$50° 36'$	$+ \log p_5$ $\bar{1}.7482$
	$II\,t'_5$	$= 236° 24'$	$II\,h'_5$ $= \bar{1}.0765 = 0.119$

And as before:

COMPLETE OSCILLATIONS.

ST. CLAIR.		ERIE.		ONTARIO.	
t.	$II\,h_3$.	t.	$II\,h_4$.	t.	$II\,h_5$.
124°	+0.517	186°	+0.213	236°	+0.119
94°	+0.448	156°	+0.184	206°	+0.103
64°	+0.259	126°	+0.107	176°	+0.060
34°	0.000	96°	0.000	146°	0.000
04°	−0.259	66°	−0.107	116°	−0.060
− 26°	−0.448	36°	−0.184	86°	−0.103
− 56°	−0.517	06°	−0.213	56°	−0.119

In the case of St. Clair there is lacking the observed oscillation to start with, but this may be closely approached by assuming, for its local supply a value of a_3 corresponding to its water-shed, and summing the effects of this with the effects from above.

In this case a_3 was finally taken as most probably about 8 000 with the same period as the Michigan-Huron supply, giving $S = 8\,000 \sin t$, where the t is reckoned on the Michigan-Huron scale.

Here then

$$III\,h'_3 = a_3 f_3 \begin{cases} 8\,000 & = 3.9031 \\ f_3 & = \bar{5}.5826 \\ III\,h'_3 & = \bar{1}.4857 = 0.306; \end{cases}$$

(log)

and the following are the

MAXIMUM EFFECTS:

Michigan-Huron time.			log
	$III\,t'_3$ = 96° 02′		
	56°		
Superior time.	$III\,t'_3$ = 40° 02′	$III\,h'_2$	= 1.4857 = 0.306
	$+R_4$ 61° 38′	= log	p_4 1.6147
	$III\,t'_4$ = 101° 40′	$III\,h'_4$	= 1.1004 = 0.126
	$+R_5$ 50° 36′	+ log	p_5 1.7482
	$III\,t'_5$ = 152° 16′	$III\,h'_5$	= 2.8486 = 0.070

And as before

COMPLETE OSCILLATIONS:

ST. CLAIR.		ERIE.		ONTARIO.	
t.	$III\,h_3$.	t.	$III\,h_4$.	t.	$III\,h_5$.
40°	+0.306	102°	+0.126	152°	+0.070
70°	+0.265	132°	+0.109	122°	+0.061
100°	+0.153	162°	+0.063	92°	+0.035
130°	0.000	192°	0.000	62°	0.000
160°	−0.153	222°	−0.063	32°	−0.035
190°	−0.265	252°	−0.109	02°	−0.061
220°	−0.306	282°	−0.126	−28°	−0.070

In this case in the place of the computed trial oscillation, the complete oscillation of St. Clair is taken to add to the effects from above.

For Erie, again subtracting the effects of the three lakes above from its mean levels, the following is left for the local observed oscillation and the computed oscillation corresponding to it.

Local observed oscillation.				Computed Trial Oscillation.		
				t	$IV\,h_4$	t
June.....	+0.545	+0.545	June.....	152	+0.500	152
May.....	+0.470	+0.415	July.....	122	+0.433	182
April.....	+0.215	+0.165	August....	92	+0.250	212
March	−0.145	−0.075	September	62	0.000	242
February .	−0.275	−0.355	October ...	32	−0.250	272
January...	−0.295	−0.490	November.	02	−0.433	302
December.	−0.395	−0.395	December .	−28	−0.500	332

And accepting this trial oscillation, there results

$$\left.\begin{aligned} 0.50 &= \overline{1}.6990 \\ f_4 &= 5.1995 \\ a_4 &= \overline{4}.4995 = 31\,580 \end{aligned}\right\} \text{ or } IV\,S = 31\,580 \sin t.$$

(log)

While plotting and comparing it is seen that the zero of t is some 6° earlier than the mid-January period, or in this case 66° must be subtracted from all values of t, to reduce them to the Superior time scale.

Hence:

Maximum effects.					Complete oscillation. Ontario.	
					t	$IV\,h_5$
Erie time.	$IV\,t'_4$	= 151° 38′			136	+0.280
		66° 00′		log	106	+0.243
Superior time.	$IV\,t'_4$	= 85° 38′	$IV\,h'_4$	$= \overline{1}.6990 = 0.500$	76	+0.140
	$+R_5$	50° 36′	$+\log p_5$	$\overline{1}.7482$	46	0.000
	$IV\,t'_5$	= 136° 14′	$IV\,h'_5$	$= \overline{1}.4472 = 0.280$	16	−0.140
					− 14	−0.243
					− 44	−0.280

Again, for Ontario, we have:

Local Observed Oscillation.				Computed Trial Oscillation.		
				t	$V\,h_5$	t
May.....	+0.665	+0.665	May.....	136	+0.665	136
April.....	+0.555	+0.640	June.....	106	+0.573	166
March	+0.130	+0.365	July.....	76	+0.338	196
February .	−0.135	+0.075	August....	46	0.000	226
January...	−0.275	−0.300	September	16	−0.333	256
December.	−0.530	−0.575	October....	−14	−0.579	286
November.	−0.665	−0.665	November.	−44	−0.665	316

From which

$$\left.\begin{aligned} 0.665 &= \overline{1}.8228 \\ f_5 &= 5.2711 \\ a_5 &= \overline{4}.5517 = 35\,620 \end{aligned}\right\} \text{ or } V\,S = 35\,620 \sin t.$$

(log)

While plotting and comparing it is seen that the zero of its t falls 14° earlier than the mid-January period, or subtracting 74° from the t of its trial oscillation brings it also upon the Superior time scale.

In going down the chain of lakes, out of these different elements of their oscillations any desired combination can be readily made up by scaling coincident values of h and taking their algebraic sums. Of these the leading combinations have been made up and shown with the observed variation of the mean levels on Fig. 4; and, indeed, it is mainly in their graphic representation that this series of movements is best followed. However, the purpose of this study has been simply the analysis of these movements, and with the equations and the illustrations that have here been given, it is thought that there are few questions that may arise in regard to variations of supply and their effects through any chain of lakes or reservoirs that may not be readily calculated.

In contrast with these relations in the lake levels it may be interesting to note briefly the character of similar relations in the Mississippi River. There, of course, the mathematical basis is wanting. Between the lakes, it matters not what energy may be stored in the flow, it is wholly absorbed through agitation in the inert mass below. Thus the sheer fall of the Niagara River makes not the slightest difference in the relation of supply to discharge, or the rise of one lake to the other; but it would certainly be a bold assumption to start with, to hold that essentially the same thing was true for the river; and even if that were the case, it is known that the linear variation of Q to h is not correct through the great range of its flood oscillations.

While, therefore, we cannot here calculate between two points what should be the retardation R, the reduction P, or the ratio of rise p, there are at some twenty or more locations along the Lower Mississippi twenty-five to forty years of daily gauge-readings from which to determine what they are; and as all these data have been worked over, again and again, along just these lines, its positive showings at least merit a comparison with the above theoretic deductions.

They may be stated as follows: First, for the ratio of reduction P, as might be expected, it is practically unity. If there is any general decrease in the magnitude of the discharge as the flood-wave passes down, it is altogether beyond the range of observation. However, p, the ratio of rise or fall, has in general a marked difference in value between different gauges. It is a well-determined constant for each reach of the river; and while in the course of a number of years it may change to some extent with a change of regimen, in the lower Missis-

sippi it is remarkably stable. Finally, R, the retardation, is also there a constant of the reach. Its value is very different from gauge to gauge, and differs greatly per mile in different parts of the river, but once determined for a given reach it seems to be practically the same there for all floods and for all time. It is, however, measured in days and hours, where the retardation of the lakes is measured in months and years; and, unlike the retardation, which is a fraction of the cycle, the retardation in the river is the same absolute time, whether the flood-wave is long or short.

From these relations of the river it might be inferred that we are dealing with something like a limiting case of a chain of lakes, as, indeed, it is well known that we are. Were the Mississippi to run down to a zero of discharge, that would be exactly what it would show, only here their areas, some 5 to 15 miles long, and from a half to a mile wide, are almost lost sight of, in comparison with the great discharge capacity that exists between them. In general, also, it is a fact that at the highest stages the greatest velocities are found in the locations of the lakes themselves; and with their functions thus reversed, they may be much more nearly taken to represent the points of outlets. It is not, however, a part of this study to consider theories of flow; and it is enough here to note that, as the character of flow through a chain of lakes corresponds to observed facts in the river, it furnishes the basis of a radical departure from the whole series of assumptions upon which hydraulic formulas have heretofore rested.

DISCUSSION.

Mr. Wisner. GEORGE Y. WISNER, M. Am. Soc. C. E.—The members of the Society who are interested in the hydraulics of the waterways of the Great Lakes are certainly deeply indebted to the authors for their very elaborate analysis of the theoretical effect of storage on flow of water in rivers.

In order that the old adage that "figures cannot lie" may hold true in all cases, it is necessary that the premises and the data from which the formulas are deduced be strictly true, otherwise the practical application of such formulas may lead to the absurd result that actual existing conditions are impossibilities.

On page 402 it is stated that the supply to any given lake from inflow, drainage, precipitation and evaporation will vary as the sine of an arc on a circle whose whole circumference represents one year; that "the discharge from the lake is, of course, determined at any time solely by the value of h, or the level of its outlet at that time"; and that "it is sufficient to take Q as made up of a constant element Q_1, the value of discharge at an assumed level, and a variable element $c\,h$, where h gives the surface at any time above or below this arbitrary level, and c is the average change in discharge there per foot of rise or fall."

These assumptions come about as near representing the actual conditions on the lake waterways for any given year as a composite photograph of a hundred persons would represent the individual characteristics of any one of the sitters.

The formulas based on these assumptions have been derived by the use of the integral calculus, which presupposes that the supply, the discharge through the outlets, and the oscillations of the lake surfaces are regular variables; while in fact they are well known to be so irregular that it is impossible to predict their variations one month in advance. From these deduced formulas it is shown (see page 363) that the ratio of the maximum supply to the maximum discharge is independent of supply. This conclusion alone is sufficient to indicate that the results derived theoretically do not conform with the facts.

The supply is made up of the algebraic sum of the inflow from other lakes, the run-off from the water-shed, the precipitation on the lake, and the evaporation from the surface of the lake. The first three of these elements are positive and the last negative, and as a general rule it may be stated that years of maximum precipitation and inflow are years of minimum evaporation, and *vice versa*. An examination of the tables of water levels of Lakes Michigan and Huron for the past 35 years, shows that the fluctuations of the levels, due to variation in the supply and discharge, at times amount to as much as 8 ins. per month

in depth over the whole lake, corresponding for a rising stage to a positive supply of about 500 000 cu. ft. per second, and for a falling stage to a negative supply of about 200 000 cu. ft. per second, making the total variation in the supply 700 000 cu. ft. per second. The area of Lakes Michigan and Huron varies but little for different stages, and, therefore, whatever theoretical change takes place in the ratio of maximum supply to maximum discharge for different seasons or stages depends on the value (c) of the increment of change in discharge for 1 ft. change in stage—a quantity which has been assumed by the authors to be a constant, or, in other words, the ratio would be nearly constant, which the tables of water levels show is not the case. Mr. Wisner.

The fundamental equation, $Q = f(h)$, in which $f(h)$ always increases and diminishes with h, is all right for reservoirs under certain conditions, but, unfortunately, these conditions do not now exist at the outlet of Lake Huron. In fact, the conditions the authors have assumed to exist are the very ones that the advocates of lake regulation are desirous of producing.

The St. Clair and Detroit Rivers, being simply connecting links between Lakes Huron and Erie, their surfaces fluctuate with the levels of the two lakes and with a slope depending upon the respective levels of each, and since the fluctuation of Lake Erie exceeds that of Lake Huron, the slope of the rivers is generally greater at the low stages of the lakes than at the high stages. A large percentage of the fall from Lake Huron to Lake St. Clair (about 5.4 ft. for mean stage) is at the outlet of Lake Huron (which at the gorge is only 750 ft. wide), and any change in this fall, due to unequal changes of levels of the respective lakes, produces very decided changes in the slope of the rapids at the outlet of Lake Huron, and a corresponding change in the volume of the discharge. The volume of discharge, therefore, not only depends on the stage at the outlet, but also on the relative elevation of the water surface of Lake St. Clair; but as the water surface of the latter follows the level of Lake Erie very closely, the discharge of Lake Huron will depend on the stage of that lake, and also on that of Lake Erie. If the level of Lake Erie be so regulated that the oscillations, aside from those due to winds, be reduced to a few inches, the area of the cross-section of the outlet of Lake Huron and the slope of the river through the gorge would depend upon the stage of water at the outlet, and the fundamental formula used by the authors would then be applicable. It needs no mathematical demonstration to establish the fact that storage is absolutely essential to any system of improvement of the lake waterways based on the regulation of lake levels. The studies which are now being made of this subject contemplate only such changes in the flow through the lake outlets as will produce the maximum benefit to commerce, by controlling the discharge at Buffalo sufficiently to maintain a nearly constant level for

Mr. Wisner.

the lake, and at the same time rendering the storage in Lakes Michigan and Huron more effective, by making the discharge from Lake Huron depend on the stage of water in those lakes. The slope of the St. Clair, under present conditions, is greatest during the low-water season of the winter, but, with Lake Erie regulated, it would be a minimum in the winter, thus giving a maximum effect to the storage capacity of Lakes Michigan and Huron.

The area of the lake reservoir system above Lake Erie is such that, for an average season, the inflow through the Detroit River would raise the level of Lake Erie 22 ft. if not discharged, and the changes in the flow, which will be produced by regulation, will be so small, compared with those due to variation of precipitation and evaporation, that no injurious results can possibly arise.

It is stated that "The balance of forces which Nature has here produced in the course of long ages is one of the most marvelous features of these lakes; and a careful contemplation of it cannot fail to convince one that an almost perfect compromise has been reached between the conflicting oscillations of lake level and outlet discharge. It is difficult to see how either could be brought nearer to absolute uniformity without a resulting departure in the other which would more than offset the gain."

The same, with equal fairness, may be said of the tidal forces, which in a long series of years have produced certain depths on the sand bars which obstruct the entrances of ocean harbors; yet where would the commerce of this country be to-day if these forces, under the guiding skill of the engineer, had not been so modified that they have produced and maintained channels of more than double the natural depths? If it is true that beneficial results cannot be produced by modifying the action of the forces of Nature, the writer sees no reason why the maritime and river and harbor engineers should not retire from business.

The author of the first part of the paper advocates placing regulating works directly in the outlets of lakes, such that the piers would contract the cross-section sufficiently to raise the lake level, and, by means of needle dams, control the oscillation of the lakes.

It is certainly a fact that, if the gorge of the outlet be diminished in width, the mean level of the lake will be raised, and it is equally true that such contraction will increase the fluctuations of the lake level and thereby augment the evil which it is wished to correct, and at the same time cause great damage to the existing structures of the lake harbors.

If the regulating works should be placed at the foot of the lake, above the outlet, the water in the gorge below would rise and fall with the change in the discharge over the fixed dam, or through the regulating dam—either of which would accomplish the purpose—and thereby carry off the surplus water at the time that the lake in its

natural condition would rise, and restrict the outflow during the remainder of the season, when, under the present conditions, the discharge through the outlet is greater than the supply. Mr. Wisner.

It is stated that "The only way the effects of diversions can be counteracted is to contract the outlets, so that the flow through them at the normal mean level shall be diminished by an amount equal to the diversion."

The result of such a remedy would be worse than the evil it is intended to correct. It is a well-known fact that vessels on the lakes are constructed with reference to maximum navigable depth in the lake channels, and any improvement tending to increase the fluctuations in these depths would be very injurious to the lake commerce.

The depth, which may be obtained and maintained through our lake waterways and thence to the sea-board, is not nearly of so much importance as that it shall be made nearly constant for the season of navigation through the channels from Lake Huron to Buffalo.

This cannot be done by any contracting works in the outlets of the lakes, but it can be accomplished by regulating works at the foot of Lake Erie, leaving the outlet unobstructed to take care of the maximum discharge, which would be approximately the same as under natural conditions. With the level of Lake Erie regulated, the discharge at the outlet of Lake Huron would vary with the stage of water in the lake, and, if it should then be found that the storage capacity of Lakes Michigan and Huron was in excess of what was needed, the fluctuations of those lakes could be reduced by enlarging the cross-section of the gorge at the head of the St. Clair River.

In the improvements, by dredging, which have heretofore been made in the St. Mary's, St. Clair and Detroit rivers, the material excavated has been carefully deposited in the streams adjacent to the cuts, for the purpose of maintaining the normal cross-sections as established by Nature, and it has been shown, theoretically, by several very elaborate reports, that these improvements could to no appreciable extent lower the slopes of these waterways; yet, in spite of these theoretical impossibilities, the water level below the locks at Sault Ste. Marie is about half a foot less than for the same stage in Lake Huron before the improvements were made, and the slope from Lake Huron to Lake Erie, for an average stage of the two lakes, is about 7 ins. less than it was previous to the deepening of these waterways.

H. W. BRINCKERHOFF, M. Am. Soc. C. E.—It seems possible that some of these calculations of lake discharge might have to be labeled as Mr. Dunn, late weather prognosticator of the first district, used to mark his prophecies—"Subject to revision by the east wind." Of course the effect of the wind on lake discharge would be temporary and perhaps slight; but the speaker remembers hearing an eye wit- Mr. Brinckerhoff.

Mr. Brinckerhoff.

ness state that the flow of the Niagara River was, in the early part of the century, once completely checked, so that for a few hours it was possible to walk across the bed of the river above the Falls. This was caused by the wind, but not because the wind blew up the river. Lake Erie is the shallowest of all the Great Lakes, and, therefore, more easily affected by the wind, which causes at Buffalo, and presumably also at the other end of the lake, quite extensive wind tides. On this particular occasion it is probable that the water in the lake was unusually low, but, in addition to that, the wind, for some time blowing strongly from the upper end of the lake toward the lower, forced out through the Niagara River all the water that it could, and then turned around and blew the other way, and lowered the remaining water so much at the eastern end of the lake that it was below the level of the bar or shoal at the head of the Niagara River, thus stopping its flow for the time.

Mr. Johnson.

J. B. JOHNSON, M. Am. Soc. C. E.—The regulation or control of the level of the Great Lakes, while apparently a small matter—and many, perhaps, have jumped to the conclusion that it is very simple—is really a serious and difficult problem, and should be approached with extreme care, and only after a long series of physical investigations. The speaker believes that such investigations are now being undertaken by the officers who are in charge of the survey of the lakes, and that a number of years will be given to the study of this subject. Active work has already begun, tending to develop all the facts that would seem to be material. This paper lays out the ground on which will have to be based all future studies for the improvement of the lakes, their harbors and their connecting channels, and is a valuable contribution to professional literature.

On the Michigan Peninsula and around the shores of Lakes Michigan and Huron on the south, the ground is so extremely sandy that the underground flow must be very large, so that these lakes receive a great quantity of water which could never be ascertained by the ordinary method of observing run-off by gauging streams.

CORRESPONDENCE.

Mr. Roberts.

THOMAS P. ROBERTS, Esq.—Captain Chittenden, on page 359, says:

"It may be accepted as a preliminary condition, to which there can be no qualification, that any alteration of natural conditions in the Great Lakes, which shall result in a material diminution of flow in the outlets during any portion of the navigation season, cannot be considered for a moment."

This bold sentence appears as a syllabus in the opening of the subject, and is well calculated to draw attention and excite discussion, for not all of his readers will agree with its sweeping conclusions.

It may be admitted that no diminution in the flow of the St. Mary's, and possibly in the Detroit River, is to be permitted during the season of navigation, *i. e.*, reducing their flow below their mean discharge, but the remark is not so fully applicable to the Niagara, which is not navigable, unless, indeed, it be shown that no contracting works to compensate for the diminished supply to Lake Ontario can be tolerated on the St. Lawrence. May it not be possible, even with the discharge of the St. Lawrence somewhat diminished, with training works to maintain its navigation unimpaired? Theoretically, at least, it does not appear to the writer to be practicable to maintain the levels of the Great Lakes at or above their mean levels without, during seasons of protracted drought, somewhat trenching upon the discharge of the outlets, yet he fancies this trenching will not actually amount to reducing their discharge to less than their extreme normal minimum flow. Their least discharge with dams should be more than their existing minimum, because the dams, by increasing the areas and capacities of the lakes, under Captain Chittenden's Fundamental Proposition I, will give to their outlets a more uniform discharge than they would have without them. However, as "we must not trust too far to keeping our pennies while buying our pies," at the same time let us not be alarmed too much concerning the outlets. If some of their water supply should happen to be shut off, it is within the range of art to compensate for such mischief.

The writer finds himself mentally noting "correct," "correct," to paragraph after paragraph of Captain Chittenden's propositions and arguments, though all the time instinctively feeling that to agree to all of them will leave the advocates of dams with not a leg to stand on, and to almost force the conclusion that nothing whatever can be done for the improvement of the Great Lakes. Captain Chittenden and Mr. Seddon deserve thanks, if for nothing else, in having demonstrated that great carefulness is to be exercised, and that only conservative ideas of lake improvement are likely to be ever realized.

Referring to one of the deductions from Proposition III, relating

Mr. Roberts. to the suggestion of storing water in Lake Michigan to make up the amount diverted for the proposed maintenance of the Chicago drainage canal, Captain Chittenden says, page 381:

"The storage of water in Lake Superior must commence by cutting off the outflow. When the stored water is run out, the total increase of flow over the normal condition will only be equal to the previous decrease. In other words, the total supply to the lakes below, upon which their mean level depends, cannot be altered a particle by storage."

Certainly the total supply to the lower lakes will not be altered a particle, but this is far from proving that the regulation of the supply from Lake Superior will not prove of benefit to Lakes Huron and Michigan. It may be found practicable to so gradually store water in Lake Superior as not to, while so doing, diminish the twin level lakes below a desirable least level. However, the amount of damage which is likely to result by reason of overflow along the shores of Lake Michigan should be known before seriously discussing the utility of that great lake as an impounding reservoir.

In the case of the improvement of a river by means of permanent dams, the engineer expects to increase its mean level, but at the same time reduce the range between its high and low stage. The only thing calling for much concern is the effect of his dams in increasing the height of the maximum rises. It cannot be said that dams at the outlets of the Great Lakes would be analogous to dams on an average river, for very likely fixed structures in the outlets of the lakes would disastrously affect the maximum levels of the lakes above them, but with a system of movable dams under proper management, the writer contends that the records of the fluctuations of the lakes during the past quarter of a century do not afford ground for alarm.

Referring to the profile of the lake oscillations, Plate XXXVI, on Lake Superior is found a conspicuous apex during the summer of 1876, so marked indeed that it becomes a matter of practical importance to know what damages would result from considering it an elevation which might hereafter be approximated, at least somewhat more frequently than in the past, if a dam were constructed on the St. Mary's River. The profile indicates, however, that for three years preceding 1876, the lowest levels of Lake Superior were above the mean level, so that the probabilities are that if an adjustable dam had been in existence during those years it would have remained down, and consequently the rise of 1876 would not have been augmented by its existence. If, in the fall of 1876, the dam had been raised, it could have held surplus water until 1879 or 1880, which (without a dam on the St. Clair River), would have assisted in maintaining Lakes Michigan and Huron. However, it is to be granted that when such large profile areas of depression as existed in the lower lakes between 1890 and 1898 are considered, it becomes manifest that something

more than storage water from Lake Superior is demanded to meet the requirements of navigation. Mr. Roberts.

For Lake Erie there appears to have been no urgent necessity for increased depth from 1875 to 1891, a period of 16 years. Let it be supposed that an adjustable dam had existed in the Niagara River during these years. It would probably have been lowered in 1882, when the indications were that the lake would not decline to mean level. It would have been partially raised in the beginning of 1889; lowered one year later early in the season, possibly without materially increasing the summer rise of 1890; again raised in the fall of 1890, and kept either altogether or partly raised, to the present season. This would have made only four maneuvers of the dam necessary in sixteen years' time, the last maneuver holding Lake Erie at, or somewhat above, its mean level for seven years, during which time it is difficult to see how the discharge of the Niagara River would have materially varied from its discharge during those years, with no dam supposed to be in existence. The summer rises on Lake Erie would have been several tenths higher with a dam than without one, but the range between high and low on the lake would, with the dam, have been greatly lessened, and a lessened range on the lake implies a more equable discharge in its outlet. Nor does the study of the profile of oscillations indicate that with adjustable dams at the outlets of any of the lakes, excepting possibly Ontario, arranged to be lowered invariably upon the advent of rises above mean lake level, would have added to their maximum rise, for the reason that in no year preceding a decided maximum on the lakes above Ontario is found, during the navigation season, a level below the mean, calling for their raising; consequently, the dams would have been down many months before the maximum floods neared the danger line. The year 1890 is an apparent exception to this rule, for Lake Erie, when a winter low water of about 1 ft. below mean level was followed six months later with a rise of 1½ ft. above mean level.

Whatever importance Captain Chittenden may attach to his own deductions from the propositions laid down by himself, it is gratifying (see page 383), if dams must be had, to find him endorsing a series of needle dams between piers extending across the Niagara, in preference to a fixed wier. The writer entertains the same preference for an adjustable structure at that point, and, in January, 1896, in a communication to Senator Brice, which was published, along with communications from others, in a report, without a document number, of a committee to the Senate, suggested pontoons between piers,

The writer never entertained the idea that the lakes could be maintained at a substantially uniform level, but believes it to be practicable to confine their range between their present mean and high-water levels. The phenomena on the lakes which will follow the

Mr. Roberts. construction of controlling works which would affect this result may indeed present novel aspects responsive to the rules laid down in the paper, but not attended with consequences, in all probability, as alarming as might be inferred by some who will grant the correctness of the propositions advanced by its authors.

Mr. Rafter. GEORGE W. RAFTER, M. Am. Soc. C. E.—The general question of the effect of reservoirs as moderators of flood flows, and the use which can be made of their water registers to determine the yield of the tributary drainage areas, has been, so far as known to the writer, more thoroughly discussed in this paper than in any other place. The authors deserve great credit for the thorough manner in which they have treated the problem. Among the more interesting discussions of the general proposition with which the writer is familiar, that of Lieutenant-General Mullins, as given in his Irrigation Manual, may be mentioned. In that discussion General Mullins refers to formulas devised by General O'Connell, of the East Indian service, which, however, have not passed under the writer's notice any further than as referred to in the "Irrigation Manual." The general problem has been discussed in other places, but the present paper must be credited with being the most complete of any discussion thus far made.

Many of the propositions of the paper are fundamentally sound, although the writer cannot but think that the methods of computation used are more cumbersome than actually necessary. It is believed that an arithmetical solution could be used with considerable economy of labor and time. Under this head, it may be pointed out that the authors' use of quantities in cubic feet per second is one of the unnecessarily cumbersome features of the computation. A reduction of the data to inches on the water-shed, and a rational use of the same, would by itself simplify the treatment greatly. It is not desired, however, to specially press this point, but rather to point out what seems to be an unsound position of the authors in regard to the desirability of considering rainfall, run-off and evaporation data.

It is conceded that the uniformity of flow characterizing the discharge of the Great Lakes is due, purely, to the equalizing effect of the vast storage on the surfaces. The position of the authors apparently is, that the effect of this storage is so great that consideration of the temporary effect of rainfall and run-off from the tributary land areas is unnecessary. It seems to the writer that this is equivalent to saying that the whole is less than the sum of all its parts. Since making, two years ago, a brief study of some of the questions discussed in the paper, it has seemed to the writer an exceedingly interesting circumstance that, notwithstanding all the discussion of outflow of the Great Lakes which has taken place, no one has thus far been able to treat the rainfall and run-off part of the problem satisfactorily.

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This is the more astonishing when is considered the number of rainfall data, applying to the basin of the Great Lakes, which are now available. A considerable number of meteorological stations have been maintained in this drainage basin by the Federal Government from about 1871, while all the States bordering on the Great Lakes have maintained State services for a number of years, thus accumulating a vast amount of meteorological data. The Lake Survey also gathered considerable useful information of this character, from about 1866 or 1867 to 1871 or 1872. To the north of the Lakes, the Dominion Government has maintained meteorological stations since about 1870. There are, therefore, fully twenty-five years' data of the rainfall of the Great Lakes Basin; and while it is true that a longer period would give more accurate means, still, inasmuch as this happens to be the kind of a problem where means are not specially important, it is believed that the data are sufficient for a rational solution. As to the proper method of treating this portion of the data, the writer would suggest the preparation of a map of the drainage basin, on which should be compiled all the rainfalls for the period covered by each record. This map should be divided into isoclimatic areas, each representing a given rainfall. With the exception of the widest portion of Lake Superior, there is no good reason for supposing that any of the lakes are so wide that the lines indicating these divisions could not extend, without material error, across the lakes, there being, thus far, absolutely no evidence derived from actual observation that the rainfall over the water areas is very materially different from that on the adjacent land areas. With such a map, and with the areas of each district carefully taken out, it would not be difficult to compute the mean rainfall for any given year over the entire drainage basin far more accurately than the run-offs through the outlets can ever be measured. Such a map may be termed an isoclimatic chart, and, with the large number of data now at hand, the sub-areas could be so apportioned that the meteorological conditions in each area would substantially correspond.

The authors have referred to the year 1895 as the one of extreme low water in all the lakes except Superior, which during that year was slowly rising. The following references to the rainfall data for the years from 1892–95, inclusive, will show not only that a fall in the lakes was inevitable during the period mentioned, but also that, on the whole, such decrease in elevation was in accordance with the clear indications of the rainfall record. As to the fact that Lake Superior was rising while the other lakes were falling, the writer at this time attempts no explanation.

Table No. 5 contains statistics of rainfall for the years 1892–1895 at a number of points either actually in or very near the Great Lakes Basin, which have been prepared for water years, each beginning with

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the month of December of the previous year, and terminating with the month of November. The water year is divided into three periods; the months from December to May, inclusive, forming what is termed a storage period; June to August, inclusive, the growing period; and September to November, inclusive, the replenishing period. Under this arrangement the water year for 1892, for instance, extends from December, 1891, to November, 1892, inclusive, and so on for the other years.

The run-offs of the growing and replenishing periods are comparatively small, and the statements of the quantities of rainfall for these periods are, therefore, omitted in the table in order to save space.

While the table contains only a few of the many records that could be cited from the Great Lakes Drainage Basin, still, inasmuch as nothing more than illustration is required, it may be considered sufficient at this time. It is easily determined that, at the stations cited, the rainfall was either very low for all the years in the period under consideration, or else that for the year 1895 it was much lower than the mean. Duluth, Minneapolis, Green Bay, Madison, Milwaukee, Chicago, Logansport, Ann Harbor, Grand Haven, Saint Ignace, Traverse City, Cleveland, Toledo, Buffalo, Winnipeg and Toronto may be cited in proof of this position.

TABLE No. 5.—RAINFALL, IN INCHES, AT VARIOUS PLACES IN OR NEAR THE GREAT LAKES BASIN.

	Storage Period, December to May (inclusive).				Total Yearly Rainfall. December to November (inclusive).				Mean of four water years.	Difference between columns 8 and 9.	Difference between columns 8 and 7.
	(1) 1892	(2) 1893	(3) 1894	(4) 1895	(5) 1892	(6) 1893	(7) 1894	(8) 1895	(9)	(10)	(11)
Pokegama Falls, Minn.	11.76	9.64	14.72	9.14	21.61	25.54	31.77	27.33	26.56	+ 0.77	— 4.44
Duluth, Minn.	17.96	11.08	19.44	6.44	32.13	21.58	31.75	23.46	27.23	— 3.77	— 8.29
Minneapolis, Minn.	13.78	12.80	15.66	7.72	39.43	28.27	23.85	22.69	28.56	— 5.87	— 1.16
Green Bay, Wis.	14.95	14.85	19.65	10.06	35.37	30.12	38.63	20.72	31.21	—10.49	—17.91
Madison, Wis.	18.89	13.37	10.96	5.54	37.12	31.94	24.80	11.99	26.46	—14.47	—12.81
Milwaukee, Wis.	18.17	15.69	15.94	10.34	34.88	31.89	29.54	23.12	29.86	— 6.74	— 6.42
Chicago, Ill.	16.03	13.93	14.48	9.58	36.25	26.96	27.94	27.28	29.61	— 2.33	— 0.66
Logansport, Ind.	27.26	24.45	21.11	9.08	45.14	37.89	33.51	24.04	35.15	—11.11	— 9.47
Ann Arbor, Mich.	14.50	20.54	16.63	8.92	30.38	38.05	26.60	19.02	28.50	— 9.49	— 7.58
Grand Haven, Mich.	16.57	19.02	19.84	10.89	30.47	34.49	34.99	21.94	30.47	— 8.53	—13.05
Marquette, Mich.	16.03	14.81	24.65	16.35	28.45	31.38	39.21	32.28	32.83	— 0.55	— 6.93
Saint Ignace, Mich.	12.39	15.61	17.80	11.66	29.12	31.86	34.43	25.23	30.16	— 4.93	— 9.20
Traverse City, Mich.	17.65	17.88	20.62	16.69	37.13	36.36	35.95	29.07	34.63	— 5.56	— 6.88
Cleveland, O.	19.84	19.09	15.28	9.29	37.65	32.00	28.60	24.79	30.76	— 5.97	— 3.81
Toledo, O.	17.77	10.17	14.93	9.23	37.00	21.90	22.94	22.58	26.10	— 3.52	— 0.36
Buffalo, N. Y.	22.62	20.65	22.47	14.17	47.87	36.52	40.79	29.25	38.61	— 9.36	—11.54
Winnipeg, Man.	6.65	8.25	8.55	8.18	19.31	23.41	18.19	17.22	19.53	— 2.31	— 0.97
Port Arthur, Ont.	8.84	8.48	8.20	8.76	21.50	22.37	22.07	22.67	22.15	— 0.52	+ 0.60
Toronto, Ont.	12.21	18.64	19.90	11.93	31.15	36.35	31.41	25.93	31.21	— 5.28	— 5.48

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The data given in Table No. 5, without being exhaustive, show that the year 1895 was, generally speaking, a year of low rainfall throughout the Great Lakes Basin, and, in view of the general accord between the rainfall record and the elevation of the lakes, the writer cannot but think that the relation between the rainfall and run-off of the Great Lakes System was, in this case at any rate, considerably closer than the authors are disposed to admit.

By way of further showing the substantial accord of the water register record of surface elevation of the Great Lakes, the rainfall and the run-off, the writer will now cite some run-off records for the years 1892–95, inclusive, of streams either tributary to the Great Lakes, or immediately adjacent thereto, the first record cited being that of the Upper Mississippi reservoirs which control a drainage area of 3 265 sq. miles in Minnesota, immediately to the west of the Lake Superior drainage basin. The rainfall of the area tributary to the Upper Mississippi reservoirs, as indicated by records kept at Leach Lake, Winnibigoshish Lake and Pokegama Falls, from 1885 until the present time, is found to be, as an average, from 24 to 26 ins. per year, and very similar in its distribution and other characteristics to the rainfall of the area tributary to Lake Superior and the northern portion of Lakes Michigan and Huron; hence the run-off of the Upper Mississippi may be taken as typical of the streams tributary to Lake Superior and the northern part of Lakes Michigan and Huron. The following tabulation shows the rainfall and run-off from these reservoirs for the period under discussion:

Year.	Mean rainfall on water-shed, in inches.	Proportion of run-off to rainfall. Per cent.	Run-off, in ins., on the water-shed.
1892	21.33	20.8	4.43
1893	25.42	14.2	3.61
1894	26.63	13.6	3.62
1895	25.11	11.1	2.79
Totals	98.49	....	14.45
Means	24.62	14.7	3.61

From this tabulation is derived the interesting fact that, during the four-year period under discussion, the mean run-off of the Upper Mississippi water-shed was only 3.61 ins. on the total water-shed, although this figure is subject to correction, because the state of the reservoirs at the beginning and ending of the four-year period is not given in the report of the United States Engineers, from which these data are taken. This correction, however, cannot be very large, because the reservoirs are so operated, generally speaking, as to empty them each year. In 1895 the run-off was only 2.79 ins. The water area of the

Mr. Rafter. reservoirs themselves is 585 sq. miles, or nearly 18% of the whole. Lakes Superior, Michigan, Huron, St. Clair and Erie, have a total water surface of 87 895 sq. miles, with a total catchment area, including the surfaces of the lakes themselves, of 265 095 sq. miles. The water surface of these several lakes is, therefore, about 33% of the entire area of the basin, or nearly double the relative area of water surface and catchment area of the Upper Mississippi reservoirs. With other conditions the same, this fact might possibly lead to a somewhat greater proportion of run-off from the Great Lakes.

Another stream in the vicinity of the Great Lakes drainage area, the Des Plaines River in Illinois, may be referred to. In 1893, the mean rainfall on the basin of this river, which has a drainage area of 633 sq. miles above the point of gauging, as determined from the mean of a large number of stations in Illinois and Wisconsin, was 26.96 ins., while the run-off for the water year was 10.14 ins.; in 1894 the rainfall for the water year was 27.94 ins., and the run-off, 7.70 ins. For the water year 1895, the rainfall was 27.28 ins. The run-off record for that year is, unfortunately, incomplete, there being no record for the months of December and January, but from what is given and from the sequence of the rainfall, it is clear that the total record for the year 1895 did not exceed 2.0 ins. In 1896, the rainfall increased to 39.58 ins.; nevertheless, the run-off for that year was only 6.69 ins.

Another stream for which run-off records have been kept, the Muskingum River in Ohio, may be referred to. This stream has a water-shed above the point of gauging of 5 828 sq. miles. The headwaters are not far from Lake Erie, and are on the dividing line between the hill country of the east and the prairie country of the Mississippi Valley. It is considered that this stream, therefore, presents conditions applicable to the run-off of Ohio streams tributary to Lake Erie. The rainfall record is the mean of those kept at Akron, Canton, Newcomerstown and Worcester, and is believed to represent fairly well the mean precipitation of the Muskingum drainage area. This record was carefully worked up by Captain Chittenden himself, and presented in his admirable report on the water supply of the proposed Erie-Ohio ship canal.

In 1892, the total rainfall for the water year was 41.74 ins., and the run-off 13.38 ins. In 1893, the rainfall was 42.36 ins., and the run-off of the water year 16.20 ins. In 1894, the rainfall was 30.50 ins., and the run-off 8.70 ins., while in 1895 the rainfall was 29.84 ins., and the run-off of the water year only 4.90 ins.

The Oatka Creek, a tributary of the Genesee River, was gauged from 1890 to 1892, inclusive, the drainage area above the point of gauging being 28 sq. miles. In 1892, with a rainfall of 41.69 ins., the run-off was 15.42 ins. The Genesee River was also gauged for a portion of 1893, and for the years 1894, 1895 and 1896, the drainage area above

the point of gauging, at Mount Morris, New York, being 1 070 sq. miles. The rainfall is the mean of several stations. For the water year of 1894, the total rainfall was 47.79 ins., and the run-off 19.38 ins.; for 1895, the rainfall was 31 ins. and the run-off 6.67 ins. In 1896, the rainfall was 40.68 ins., and the run-off 12.80 ins. Hemlock Lake, another tributary of the Genesee River, with a drainage area of about 42 sq. miles, gave, with a rainfall in 1880 of 21.99 ins., a run-off of 3.4 ins. Mr. Rafter.

So far as known to the writer, the foregoing streams—either tributary or adjacent to the Great Lakes area—represent, with the exception of Skaneateles Lake, everything of which gaugings have been made. The data cited are sufficient to show that for the years 1892–95, inclusive, the run-off of the streams tributary to the Great Lakes was in accord with the water register and rainfall record. These records also show that the run-off of the land areas tributary to the Great Lakes is, in years of low rainfall, very small. Assuming that in such years the evaporation is equal to the rainfall, although probably it sometimes exceeds the latter; it is discovered at once why the land contribution to the Great Lakes has in such years so little effect. It accordingly follows that in years of low run-off from the land areas, the stage of the Great Lakes is almost entirely controlled by the inflow from above. This statement, however, does not apply to Lake Superior, which has no tributary reservoir.

In considering the water surface evaporation data, the writer must admit that the material at hand is extremely unsatisfactory. It is believed, however, that with the present state of knowledge concerning the laws of evaporation, a few years' observation at a number of points about the Great Lakes would furnish sufficient data to solve this part of the problem rationally.

Summarizing the whole matter, the writer's position is, that while the authors have handled admirably the general question of the Great Lakes storage reservoirs as moderators of the run-off, that they are still somewhat lame in assuming that their discussion of the problem is in reality complete. The more rational view is that it is desirable to correlate all the phenomena, in order to reach a complete solution.

JAMES A. SEDDON, M. Am. Soc. C. E.—It is not altogether clear just what Mr. Wisner takes exception to in the mathematical analysis, for he can hardly have so misunderstood its scope as to suppose that it was restricted simply to the annual cycle. The general method of representing supply is that of superimposed sine curves; the equations of two cycles in particular were worked out, viz., that for the n years, and the yearly cycle, and with these it was also noted that for any additional eccentricity of the seasons, similar terms would follow. There is no form of reciprocal variation that cannot be so expressed except at an arbitrary discontinuity, and, to cover the whole matter, the equation for this discontinuity was also given. In the applica- Mr. Seddon.

Mr. Seddon.

tion of this, then, it would certainly have been interesting if Mr. Wisner had only undertaken to give some of those existing conditions that it proved to be impossible.

That in the application of this analysis to a chain of lakes only a single cycle was carried through in no way prevented any one from taking all others; it simply marked the way for any and all of them; while finally, in the actual example treating the mean oscillations of the Great Lakes, it was, of course, simply an interesting fact that the single annual cycle was about all that was left in these twenty-five year means. Mr. Wisner did not object to this because it did not fit the data well enough, but because it fitted the data too well.

In regard to the discharge element, it was not intended in the first place, nor is it here proposed, to enter into a general discussion of it. Those who have given any study to the great masses of such data collected will appreciate the fact that the assumptions made are sufficiently accurate for the problem; and beyond that there is no end to controversies and criticisms in it. However, Mr. Wisner's point in regard to the variation of c at the Huron outlet would be well taken if his assumptions were correct; though even in that case, if values were assigned to this variation and its effects calculated through, they would be found to be hardly worth noting.

This assumption, however, to say the least, is a peculiarly bold one. It takes only a glance at the total oscillations (Fig. 4) to show that the St. Clair level follows the Erie level very closely, but to say that the Erie level is the cause of this is quite another thing, since, for that matter, it follows the Ontario level even closer. Indeed, the St. Clair levels in this case, are wholly calculated from the probable supply of its own basin, and the effects down from Huron on it, and they have not the faintest trace of an effect up from the Erie levels in them. True, the data of St. Clair are notably deficient, and, while there has been nothing as yet published to warrant this step in Mr. Wisner's assumption, still it may, perhaps, be possible that on occasions some trace of an effect up from Erie may be detected in the St. Clair levels. To carry this up again through the St. Clair River, however, and to assume that it has a sufficient magnitude at the end to materially affect the discharge from Huron through the gorge at its head, is certainly to follow up a supposition rather further than seems wise. Indeed if Mr. Wisner will not take a caution here, he will find that Nature has an especially blunt way of her own in telling those who build on such assumptions that they are mistaken.

In regard to Mr. Rafter's suggestion of arithmetical methods, it is certainly simple enough in this way to closely approximate the effect of a variable supply on any lake, beginning with an assumed level. The only difficulty is in assuming the right level. It is true that with a given oscillation of supply, Nature might originally start from any

assumed level, but it would come in years to a balanced oscillation of the surface that had but one given value for the surface level at a given time, and long before Mr. Rafter had figured through this process he would be glad enough to drop arithmetic, and take up the methods of the calculus which determine the whole matter at once. Mr. Seddon.

Simply stating, however, the questions in such an analysis, say, of the twenty-five years means of the Great Lakes, involves a cumulative series of local effects transmitted down on levels that are everywhere conditioned by the ultimate balance of supply and discharge, and that can not be assumed arbitrarily. Considering the number of independent elements that go to make up these oscillations, it is certainly expecting a good deal to calculate them all, with the variations of each through the whole cycle in much less than the computations given, pages 420 to 425, where it is all worked through in the fullest detail. While, of course, it is in general a safe criticism to suggest that any method might be simplified, it is hardly a fair one unless at the same time it is also shown how.

Col. H. M. Chittenden.—In closing this discussion, the author of the first part of the paper will consider briefly the following points: Method of investigation; conclusions arrived at; character of works required. Col. Chittenden

The suggestions offered by Mr. Rafter were particularly pleasing to the author, because they present in minute detail the very programme actually followed by him until it proved impracticable. The attempt was made, with great labor and perseverance, to reach a satisfactory solution of the reservoir problem of the Great Lakes by commencing with the primary elements of S, viz., rainfall, run-off and evaporation. The records of rainfall were obtained from nearly every point in the St. Lawrence Basin where they have been officially kept during all or a portion of the twenty-five years from 1871 to 1895. Careful studies of the subject of run-off were made from the very records Mr. Rafter refers to, viz., the upper Mississippi, the Des Plaines, and the Muskingum Rivers. The subject of evaporation was considered in the light of the best available data. Repeated efforts were made, both for single years and for the means of longer periods, to harmonize these data with their effects as shown in the rise and fall of the lakes and in the flow of the outlets. The result in every case was failure. It was not possible to satisfy, within any admissible approximation, the following equation which we know must be a true one:

$$R_a + R_o - E \pm S_t = D,*$$

* While pursuing this line of investigation the author used depth in feet upon land area and water surface, as Mr. Rafter suggests. In the more general discussion, however, this was not found practicable. If was often necessary to use the same quantity with reference to different lakes and water-sheds, as where Q is taken with reference to the lakes above and below the outlet to which it refers. Moreover, the cubic foot being the unit almost universally used in American river engineering, it was thought best, for the purposes of comparison, to use it here. In the mathematical analysis the character of the unit was immaterial, as the symbols dealt with were general in character.

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R_a represents precipitation upon the surface of the lake;

R_o represents run-off from the land area, and includes, not only the discharge of the streams. but all subterranean flow into the the lakes of whatever character;

E represents evaporation from the surface of the lake;

S_t represents storage, and is to be considered as negative with a rising lake and positive with a falling lake, and

D represents the discharge of the outlet.

The author became thoroughly convinced that data in regard to R_a, R_o, and E in the basin of the Great Lakes are, and must ever remain, too uncertain for use in precise work. The vastness of the areas considered; the marked variations in rainfall at different points not so very far apart; the possibility that large quantities of water find their way to the lakes, as Professor Johnson suggests, by channels which can never be reached and measured; the certainty that there is an immense but indefinite body of water in the land about the lakes which flows out as the levels fall and in as they rise; and, finally, the great uncertainty in regard to evaporation;—all these drawbacks may well discourage him who tries to investigate the reservoir problem of the lakes by approaching it from this direction.

Having failed to reach any satisfactory result on these lines, the plan was adopted of studying S, not from its component parts, but from its effects, as shown in the rise and fall of the lake surfaces and in the flow of the outlets. Here, it was felt, was the nearest approach to precise data that past or future records could be expected to yield. The authors believe that this is the truly rational method of treating the problem, and that results of positive value are to be expected from it.

This is far from attempting to discredit efforts to obtain a better knowledge of R_a, R_o and E. It will always be a matter of scientific interest to reach a nearer solution of the above equation than is now possible, and assistance of great value could probably be derived from such data in any actual regulation of the lake levels.

It is hardly worth while to note in this connection Mr. Wisner's observation that the authors used integral calculus in their work and therefore had to assume a regularity of variation which is never exemplified in the actual records of lake levels. If Mr. Wisner had examined this study more closely, he would not only have avoided a misquotation of the authors, but would have seen that their method may be applied to the most irregular phenomena which the fluctuation of lake levels presents. Nowhere do the authors say, or even imply, that "the supply to any given lake from inflow, drainage, precipitation and evaporation will vary as the sine of an arc," etc. On the contrary, Mr. Seddon (page 402) says that "the actual sup-

ply in any given year can never be exactly represented by any regular variation." But by varying the length of the cycle, the most divergent cases may be studied, from the sudden rise in level due to a heavy and general rain, to those long periods of change covering several years, which the author has referred to as the cyclic fluctuation. The limitations of space made it impossible to put in a single paper a detailed explanation of the various applications of the general formula; but the method itself was indicated with sufficient clearness to be easily followed. Col.Chittenden

Mr. Wisner challenges the correctness of the statement that $\frac{Q\ (\text{maximum})}{S\ (\text{maximum})}$ is independent of S, and in so doing again misquotes the author. What is actually said (page 363) is that, "for a given cycle, R is a function of area and outlet alone, and independent of S; P also is a function of area and outlet alone," etc. This proposition is rigidly true. For a given regular cycle of change in S, whether of five days or five years, and with fixed conditions as to area and outlet, P and R will remain constant whether S be small or great.

Mr. Wisner takes exception to the author's deduction that the forces of Nature have done so much for the navigation of the Great Lakes as to make it exceedingly doubtful to what extent their action may be modified with impunity; and he holds that a general application of this principle would deprive the river and harbor engineer of his *raison d'être*. This dire foreboding is effectually dispelled by the very example which Mr. Wisner (unfortunately for his argument) has selected. The forces of Nature in tidal channels have generally been obstructive in their action; whereas, in the case cited by the author they have been exactly the reverse. Did these forces everywhere do for navigation what they have done in the Great Lakes, the work of the river and harbor engineer would indeed be greatly curtailed in scope.

The only point in Mr. Wisner's discussion which may possibly assail effectually any of the author's views is where he questions the assumption that the value of Q is "determined at any time solely by the value of h." He holds that this is not true of the outlet of Michigan-Huron—that at the lower stages of this lake the slope in the outlet is frequently greater than at the higher stages; and that, owing to this increased slope, the discharge at the lower stages is greater than at the higher. This may or may not be true. The author could not find sufficient evidence of its truth to justify him in applying a different rule to the discharge of this outlet from that of any other. He is, nevertheless, entirely open to conviction and will welcome a final determination of the matter. Those charged with this determination, however, should bear in mind that the engineering profession will be satisfied only with the most complete and convincing data. No partial or incomplete series of measurements, nor any, in fact, that fail to em-

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brace simultaneous records of discharge and stage all along the connecting link between Huron and Erie, and for every phase of the relative levels of these lakes, will be accepted as conclusive.

In regard to the effect upon slope of deepening and narrowing the channel of any stream while the area of cross-section remains the same, Mr. Wisner conveys the idea that he entertains more advanced views than other engineers. Whatever may be the "elaborate reports" to which he refers, the author recalls at least one such report, written by perhaps the ablest living authority upon the Great Lakes among Government engineers, in which the argument is enforced with great emphasis, that the effect of such a change must inevitably be to flatten the slope, and, in the case of the channels between the lakes, to draw down the levels of the lakes above. In order that no erroneous inference as to the author's position upon this point may be drawn from Mr. Wisner's remarks, the following quotation is given from page 381, where the author discusses the means of preventing the lowering of lake levels from the above causes :

"In superficial area the contraction would have to be greater than the previous enlargement, for the whole change thus made in the cross-section is in the direction of deepening and narrowing the outlet; and a deep and narrow channel will carry more water than a wide and shallow one of the same area of cross-section."

Passing now from methods to results, the author is surprised that Colonel Roberts should have found anything in the present paper to justify him in saying that, while generally agreeing with its propositions, he all the time instinctively feels that:

"To agree with all of them will leave the advocates of dams with not a leg to stand on, and to almost force the conclusion that nothing whatever can be done for the improvement of the Great Lakes."

This is certainly a very different view from that entertained by the authors themselves. If they have succeeded in correcting some fallacious ideas in regard to the practicability of eliminating the annual fluctuations in the levels of the lakes; of compensating by storage in Superior for a permanent diversion from Michigan at Chicago, and of establishing a satisfactory automatic regulation of levels in Erie by a long weir at Buffalo; they have not in any sense represented the problem of improving the navigation of the lakes by means of regulating works in the outlets as an impracticable one. In order to clear up any doubt upon this point, the conclusions indicated by their investigations may here be summarized :

(1) It is neither practicable nor desirable to restrict the range of annual oscillations of the lake levels. It is the storage represented by these oscillations that furnishes the key to the whole system of lake navigation, for without it the flow in the outlets could not be maintained at the point necessary for extensive navigation.

(2) It is practicable to eliminate the cyclic fluctuations, and to

keep the levels of the lakes from ever falling below a fixed minimum plane. This would be in itself a vast gain. If lake carriers may know for a certainty that they can always have a fixed minimum depth through the connecting channels and in the harbors, and that there will not again come those periods when the mean levels will go down and down for a series of years, surely they will be relieved of one great cause of complaint. While it is probable that any practicable attainment of this end will result in an occasional increase of the annual fluctuations, there is no apparent reason why this should be objectionable, unless such increase should reach the point of causing damages. It could be utilized to swell the flow in the outlets during the navigation season and to hold it at a uniform point during that time; for again, let it be remembered, increased fluctuation of level in the lakes is a less serious matter than increased variation of discharge in the navigable outlets, particularly in that between Huron and Erie. Col. Chittenden

(3) It is entirely practicable to raise the mean levels of all the lakes, whereby an improvement of harbor and channel depths can be accomplished throughout the system. The point of difficulty in this connection will undoubtedly be so to construct the works as not to increase slopes in the channels to such a point as seriously to interfere with navigation or to necessitate the use of locks.

(4) It is utterly impracticable to prevent by means of storage in Superior a permanent lowering of mean level in the system below, which must result from a permanent diversion of supply at Chicago. The only way to prevent such a result will be to restrict the discharge in all the outlets below the points of diversion, so that at the normal mean level, it shall be equal to the previous normal mean flow less the diversion. "Regulating works" at Buffalo may, as one writer suggests, be sufficient to prevent a permanent drop in the mean level of Michigan-Huron, due to diversion at Chicago. The author does not believe it. Neither can such works have any effect upon the mean level of Ontario.

This leads to the final topic embraced in this discussion—the character of the works required for the regulation of levels of the Great Lakes. Since the completion of his paper last April, the author has had considerable correspondence and discussion upon this feature of the subject. Nothing has developed to shake his conviction that any adequate regulation of lake levels should be by means of works under the control of human agencies, as originally suggested by Colonel Roberts. A fixed work at the outlet of Erie which should give a variation of discharge of, say, 60 000 second-feet for a variation in level of say 6 ins. cannot but be disastrous, or at least very injurious, to navigation in Niagara, owing to the frequent and sudden fluctuations of level at the foot of Erie from causes in no way connected with the problem of storage in the lakes. Far from the author's plans being

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"subject to revision by the east wind," as Mr. Brinckerhoff suggests, it is the plan of the advocates of the long weir and of automatic regulation that will have to be so revised.

If the author were rewriting his paper he would modify his treatment upon pages 369 and 382 by introducing the word "fixed" before "works," and by adding a third class of works, those, namely, which are not fixed in character, but subject to direct control. He holds that there is nothing in the present state of knowledge of the physics of the Great Lakes to warrant the sweeping assertion that contracting works in the outlets may not be so constructed, as to secure all the ends above proposed without injuring the navigable condition of the channels.

The problem of improving the navigation of the Great Lakes by means of regulating works at or in the outlets is not by any means the easy and simple problem which some of the earlier expressions of the advocates of regulation would lead us to suppose. It need occasion neither surprise nor disappointment if expected results from such works are not at first fully realized. The development of such a system will very likely be tentative in character, modified by experience with works partially built, and brought to final success, not by any single stroke of engineering genius, but by the gradual contributions of many years and many minds.